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Dedication

To my mother, whose kindness and wisdom never ceases to amaze me.

Abstract

This dissertation examines how firms manage the unfavorable impact of information asymmetry - be it not knowing payoff relevant characteristics of the employee or the inability to observe the actions taken by the employee. While firms develop a multitude of mechanisms to mitigate their impact, they aren't always able to completely eliminate them. The first essay studies the advantages and disadvantages of frequent performance reviews over intermittent performance reviews in situations where firms are unable to observe payoff relevant characteristics. Performance reviews are one of the tools used by firms to weed out ill-matched employees. How often they should be held has been the subject of a lot of debate recently. The key trade-off is the timing of information arrival and the amount of information. In frequent reviews managers learn about the employees sooner, but in intermittent reviews they have more information to go by. Each review regime performs better under different conditions. Thus, a priori frequent reviews do not always achieve better results than intermittent reviews. The second essay studies the problem of motivating effort when the employee has to plan and execute the project in the presence of limited liability. The employer is unable to observe the effort involved in information acquisition and the information itself giving rise to moral hazard. It is shown that limited liability induces a risk neutral agent to take excessive/insufficient risk for different parameter values. Thus, limited liability constraints not only raise the cost of contracting, they also change the level of risk taking. The third essay extends the model in the second essay to two periods. It is shown that the time of contracting affects the feasibility of the first-best solution.

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Chapter 1

Introduction

One of the ubiquitous features of the employee-employer relationship is information asymmetry. Firms delegate tasks to employees. Sometimes they are unaware of payoff relevant characteristics of the employee and sometimes they are unable to monitor or observe the actions taken by the employee. Often, it is a combination of the two. Over time, most organizations develop mechanisms to mitigate the unfavorable impact of the information asymmetry on output and profit. However, these mechanisms are constantly evolving, as is our understanding of them.

Economists have long studied these phenomena and the mechanisms used by firms to mitigate them. The vast literature on adverse selection and moral hazard is a testament to the range and scope of the issues firms and employees face. This dissertation seeks to add to that body of knowledge by addressing two issues faced by managers under information asymmetry. The first, studied in chapter 2, pertains to a recent debate in the personnel management literature regarding the impact of the frequency of performance reviews. The second, addressed in chapter 3, studies the optimal contract when the employee is required to perform the twin tasks of information acquisition and project selection.

Firms can rarely claim that they know all the payoff relevant characteristics of their employees. Performance reviews are the most common tool used by firms in personnel management. These reviews are meant to separate the high performers from the low performers and hence refine the employee pool. In 2012, Adobe, the American software company employing 13,500 employees worldwide, announced that it would no longer conduct annual performance reviews. Instead, the firm instituted the system of monthly "check ins". While monthly check ins have some features that are different from annual performance reviews, the important difference is the frequency of the review. More frequent reviews are meant to reveal more information to the employer in a timely manner and hence enhance the efficiency of the workforce. However, a key trade-off that is missing from this argument is the reduction in the amount of information that stems from more frequent reviewing. The fewer the number of decisions on which the review is based, the less information the employer has to help him decide whether or not to retain the employee. Chapter 2 seeks to illustrate this point.

I set up a simple model where a firm is faced with a pool of employees, some of whom are ill-matched with their jobs, but the firm is unable to distinguish them from the good matches. As they work with the firm, their decisions provide imperfect information about their fit or their underlying characteristics. The purpose of performance reviews is to use this information to draw inferences about the unknown characteristics of the employee and weed out the ill-matched ones. I compare two performance review regimes - intermittent and frequent. The two regimes are distinguished by how many times the employee makes decisions on behalf of the firm before the firm conducts a performance review. When employees make more decisions, more information is known to the firm at the time of review. On the other hand, the longer an ill-matched employee works for the firm the higher the losses to the firm from bad decisions. I compare the efficacy of intermittent and frequent reviews.

I find that intermittent reviews only support two kinds of equilibria - equilibria where each type of employee picks their preferred action and the firm knows exactly which type it is facing and equilibria where the employees pick the same actions irrespective of type and the firm learns nothing. Frequent reviews support additional equilibria wherein the ill-matched employees mimic good employees for some decisions and not for others. The firm learns a little about the type of the employee, but not enough to distinguish the two types perfectly. This is the first weakness of frequent reviews. The second kind of weakness that reviews might have is where employees take the same action irrespective of type and the firm learns nothing. This occurs in both review regimes. In this case, performance reviews fail to achieve their goal. When this situation arises, it turns out that nature of the task determines which review regime is more effective. If the desirable action is more informative, then intermittent reviews are more likely to succeed in weeding out bad matches. If the undesirable action is more informative, then the opposite is true. In sum, I argue that frequent reviews are by no means prima facie superior to intermittent reviews. In fact, under certain conditions intermittent reviews may fare better. If we add to that the fact that more frequent reviews usually cost more, it becomes less obvious that firms want to adopt them indiscriminately.

Chapter 3 of this dissertation looks at a different problem faced by firms. Information acquisition and project selection are two tasks that are often delegated to the same employee. For example, whenever the employee needs expertise for executing projects and acquiring expertise is costly, it is often the same employee who is responsible for acquiring expertise *and* executing the project. Previous work suggests that in this environment moral hazard assumes a unique dimension. The firm wishes to delegate two interrelated tasks, rather than one - learning new information and using the information correctly. The first task is costly to the executive and the second task is not. Earlier papers have shown that this problem suffers from twin moral hazard - in the costly as well as the costless task. The latter is surprising and creates an interesting set of issues, which I attempt to address with this work. Motivating the first-best outcome may not always be possible. However, most of the work on this problem so far assumes that the principal and agent have different attitudes to risk. It is, however, unclear whether this result is driven by the risk aversion of the agent or because the principal needs to motivate the agent to perform two interrelated tasks. I address this issue by showing that it is possible implement the first-best outcome if the principal and agent are both risk neutral.

The focus of the chapter, however, is on the impact of having a limited liability constraint for the agent. In most situations in which the agent assumes decision making responsibilities on behalf of the principal, the agent has limited liability. I study the above problem in the presence of limited liability (and risk neutrality of principal and agent) and find that there may not exist a contract that achieves first best. In fact, the agent often takes excessive/insufficient risk. With this result, I add to a body of literature that argues that limited liability constraints not only make contracting expensive, they alter risk taking incentives. My analysis is novel because I observe that risk taking may increase *or* decrease under the optimal contract. This is a departure from earlier work that shows that limited liability constraints increase risk taking. I then argue that for some parameter values the optimal contract is concave. It induces the agent to behave as if he were risk averse and take insufficient risk. For other parameter values the optimal contract is convex and induces the agent to behave as if he were risk loving and take excessive risk. The trade-off is between motivating information acquisition and use and reducing costs in the presence of limited liability which tends to raise contract costs for the principal.

These results suggest that excessive/insufficient risk-taking is even more likely than was previously shown to be true by the information acquisition and use literature. Lambert (1986) shows that when the agent is risk averse and there is no limited liability, the optimal contract leads to excessive/insufficient risk taking. I show that when the agent is risk neutral and there *is* limited liability there is excessive/insufficient risk taking. This seems to indicate that excessive/insufficient is not an archetype of the difference in risk attitude, but a highly likely outcome in this environment.

Most employee-employer relationships involve repeated interactions over time. In some situations this can make it easier to implement the full information outcome. However, there are many ways in which this repeated interaction can be organized. In chapter 4, I study the problem of implementing the first-best outcome when the task described in chapter 3 is delegated to the agent for two periods rather than one. I compare three contracting regimes - period by period contracting, date-0 contracting with no recontracting and date-0 contracting with recontracting. I restrict my attention to the conditions under which the first best can be implemented. Allowing the agent to recontract means allowing him to access his outside option at date-1.

The key take away from this chapter is that certain contracting regimes raise the expected payoff to the agent without the actual outside options being higher. This raises the upper bound for the set of parameters under which the first best can be implemented. The choice of contracting regime depends on the trade off between the minimum expected payoff under the participation constraint and the loss of payoff induced by not using a linear contract.

Chapter 2

Impact of timing of performance reviews

2.1 Introduction and motivation

Performance reviews are the most widespread tool used by organizations for performance management. Most private firms as well as government agencies hold an annual employee review. For that matter, elections can be thought of as a review of the performance of an elected representative at the end of a designated term. In 2012, Adobe announced that it was replacing its annual performance review system with "frequent check-ins". The reason for this policy change was to give managers a chance to monitor employee performance in a timely manner rather than annually. Whether this system is more effective than the previous one remains to be seen, but it does raise some important and interesting questions about the how the frequency of reviews might matter, especially in environments where the actions of employees are not perfectly observed. The quality of information available to managers would be significantly different under different review regimes. Further, the nature of the job and the industry within which the firm operates may affect the efficacy of these reviews. In this chapter, I attempt to shed light on some of these issues.

Consider the problem of an agent working for a firm. His job is to make decisions on behalf of the principal over time. At the end of a designated term (usually a year[1]) his performance is reviewed. The outcome of this review is often linked to the agent's incentives to perform on the job. These incentives can be explicit - in the form of wages and remuneration, or implicit - in the form of future prospects in the firm or outside options. In this chapter, I focus on implicit incentives, in particular the agent's outside options if he leaves the firm,. If, for instance, the agent's preferred actions[2] do not align with the principal's preferred action, can the firm always identify and eliminate the ill-matched agent during the review given that different types of agents may have different outside options? Further, does intermittent review, rather than frequent review, align the incentives of the principal and agent more effectively? I find that the answer to these questions depends on the quality of information about the actions chosen, and the relative importance to the agent of being kept on rather than taking his preferred action in every period. Hence, it is not clear that one review regime outperforms the other. The specifics of the environment matter a great deal.

I set up a model where (payoff relevant) information about the agent is hidden from the principal. The agent chooses actions in multiple periods. His actions are not observed, but they generate informative public signals. The principal observes the signals and updates his beliefs about which kind of agent he is facing. Based on these beliefs the principal either keeps or fires the agent and I call this a review. Thus, review is a binary decision. Situations in which the reviews lead to discretionary pay appraisals if the agent is retained are not addressed by this chapter. I study two review regimes - intermittent and frequent. In intermittent review regimes review happens once every two periods. In frequent review regimes review happens after every period. I compare the efficacy of the two.

In order to make a meaningful comparison, I describe what I mean by failure of

[1]In a survey, the Society of Human Resource Management (SHRM) find that 72% of their sample of firms has an annual performance review process in place.

[2]Equivalently, the action least costly to him

the review. The primary objective of employee reviews is to weed out employees who are not a good match for the firm in the sense that their preferences are not the same as those of the firm. Thus, if one review regime fails to single out the ill-matched candidate in a situation where the other one would, then the second regime can reasonably be said to be superior for that situation. Equilibria in which both well-matched and ill-matched employees choose the same actions every time it is their turn to choose are referred to as pooling equilibria. I define these formally in section 3 and show that in a pooling equilibrium the firm doesn't learn anything from publicly observed signals of employee performance and is unable to distinguish between well-matched and ill-matched employees. Thus, a pooling equilibrium is a failure of the review process. In this chapter, if the equilibrium under one review regime is a pooling equilibrium whereas it wouldn't be under another regime, I conclude that the latter regime is superior.

Pooling equilibria arise when outside options are low. The higher the outside option, the less concerned the agent is about continuing his career in the firm. Further, I find that whether intermittent or frequent reviews will serve the firm better depends on the quality of the information known to the firm.

The second, more subtle, kind of failure arises when reviews are less dependable. By less dependable I mean that the probability of weeding out the ill-matched employee is larger than in a pooling equilibrium, but smaller than an equilibrium where each agent chooses his preferred action every time he is called upon to make a decision. I show that this kind of failure does not arise if the principal chooses to review the agent intermittently. However, it is widespread if there are frequent reviews. The reason for this is that the principal bases his decision in the review on signals generated by only one action rather than two. The ill-matched agent takes advantage of this fact and mimics the good type *just* enough to be retained during the review.

No review regime is unconditionally better than the other in this model.

The rest of the chapter is organized as follows. I review the literature in section 2. Section 3 describes and analyzes two models - the model with intermittent review and the model with frequent reviews. The latter half of the section compares the outcomes under the two review regimes. Section 4 concludes and discusses the implications of these results.

2.2 Relationship to literature

It has long been known that time matters a great deal in environments with hidden information about the agent. Principals observe the performance of the agent over time and this allows them to learn relevant information about the agent from his actions (Radner, 1981). Holmstrom (1999) argues that in dynamic contracting problems time is important for another reason. When today's performance is linked to future payoffs, the agent has an incentive to manipulate what the principal learns about him from his actions, sometimes even by sacrificing current payoffs. While learning about the agent's ability over time makes it easier to solve the incentive problems that arise as a result of the hidden information, these future payoffs or "career concerns" can make it easier *or* harder to achieve efficiency depending on how the preferences of the agent align with those of the principal. These career concerns or *implicit* incentives (elections or promotions) are particularly important in the government sector where *explicit* incentives (wages) are often weak Dewatripont et al. (1999b).

There is significant empirical evidence that supports the theoretical literature on the importance of career concerns. For instance, it has been shown that the performance of CEOs is strongly correlated with the probability of serving on the board of the firm after retirement (Brickley et al., 1999). Similarly, Gibbons and Murphy (1992) argue that since agents care about future career concerns and these concerns weaken over time as the agent's retirement draws closer, explicit compensation can be low at the beginning of the career, but has to be high at the end of the agent's career. They find empirical support for this theory by studying CEO compensation and stock performances.

In most firms there is a fixed time between subsequent performance reviews that is known to the employee and the reviewer. Elections are also held (barring some exceptions) after fixed periods. Knowing when one's performance will be evaluated may distort the incentives of the agent compared to a world where there was no fixed time of review, thus potentially keeping the review from being effective. For instance, Downs and Rocke (1994) argue that if the agent is choosing from risky options, then a poorly performing agent might have the incentive to pick the riskier option close to an upcoming review to make a last ditch attempt to save his job. They call this "gambling for resurrection". Thus, the presence of reviews raises several interesting issues. It is in the principal's interest to know how when and how often he should review the agent.

One strand of research that directly addresses the distortions in decision making that arise due to reviews in the context of democratically elected officials. Electoral cycles in macroeconomic policy have been studied extensively. Some of the earliest theoretical work (Nordhaus (1975); MacRae (1977) etc) posits that politicians will exploit the trade-off offered by the Phillips curve before the elections to decrease unemployment by increasing inflation. The assumptions and therefore conclusions of these models were, however, called into question. Sophisticated voters would foil any attempts by the incumbent to sub optimally inflate just before the election. These models, often referred to as adaptive expectations models, were then replaced by rational expectations models Rogoff (1990); Rogoff and Sibert (1988). Electoral cycles in budgetary variables are generated by different mechanisms - most often information asymmetry. For instance, in Rogoff (1990) electoral cycles are generated by an equilibrium signaling process. Voters learn about the government's competence by a one period lag and they care about competence while voting. Governments then have a reason to exploit the information asymmetry and end up generating electoral cycles in taxes.

There is a considerable body of literature examining the empirical evidence for political business as well as budgetary cycles. The empirical evidence for political business cycles is mixed, but the evidence on political budgetary cycles is rather robust (see Alesina et al. (1989) for a detailed discussion of the evidence).

In this paper I study the efficacy of the review process for varying levels of outside options in an environment with hidden information. I argue that low outside options of employees can derail the review process irrespective of how frequently reviews are held. In addition, frequent reviews allow ill-matched employees to escape detection due to the fact that the principal's decision is based on fewer signals. Under some conditions intermittent reviewing outperforms frequent reviewing, while the reverse may be true under different conditions.

2.3 Model

2.3.1 Players, preferences and information

There are two players - an agent (a) and a principal (p). The agent makes decisions on behalf of the principal. Every time the agent is called upon to act he chooses an action from the set $\mathcal{A} = \{X, Y\}$. The agent can be of two types - "type x" and "type y", with probability α and $1-\alpha$ respectively where $\alpha \in (0,1)$. Type x's most preferred action is X and his outside option, in case he no longer works for the principal, is τ_x. Type y's most preferred action is Y and his outside option is τ_y. Types are private information.

Every time the principal moves he chooses an action from the set $\mathcal{P} = \{k, f\}$, where k stands for "keep" and f stands for "fire". Thus, performance reviews are simple; at the designated time the principal reviews whatever he knows of the agent's work and decide whether to keep him or fire him. If the agent is fired then nature picks a "replacement" agent (RA) who is type x with probability α and type y with probability $1-\alpha$.

The per period utility function of the type i agent, $i \in \{x, y\}$, is given by $u_i(c) \in \mathbb{R}_{++}$, $c \in \mathcal{A}$ and the per period utility function of the principal is given by $v(c) \in \mathbb{R}_{++}$. Type x prefers action X, so $u_x(X) > u_x(Y)$. Type y prefers Y so $u_y(Y) > u_y(X)$. I assume that $v(X) > v(Y)$, that is, the principal prefers action X to Y. Utility is additively separable over time. There is no discounting.

Pre-review actions are not observed, but every action generates a signal that is publicly observed immediately before the review. The probability of observing a signal depends on the action chosen and the period in which the action is chosen but not on the type of the agent choosing the action. The signal space is $\mathcal{S} = \{S_X, S_Y\}$. Let $a_t \in \mathcal{A}$ be the action chosen by the agent in period t and let $s_t \in \mathcal{S}$ be the signal generated by a_t. Then

$$P(s_t = S_X | a_t = X) = \theta_X^t \quad \text{and} \quad P(s_t = S_Y | a_t = Y) = \theta_Y^t$$

I assume throughout that

(i) For all $m \in \{X, Y\}$ and $n \in \{1, 2\}$, $\frac{1}{2} < \theta_m^n < 1$

(ii) For every $a_t \in \mathcal{A}$, s_t's are independent.

Thus, S_X and S_Y denote the signals observed with the highest probability if the agent chooses actions X and Y respectively. Further, signals are informative and independent.

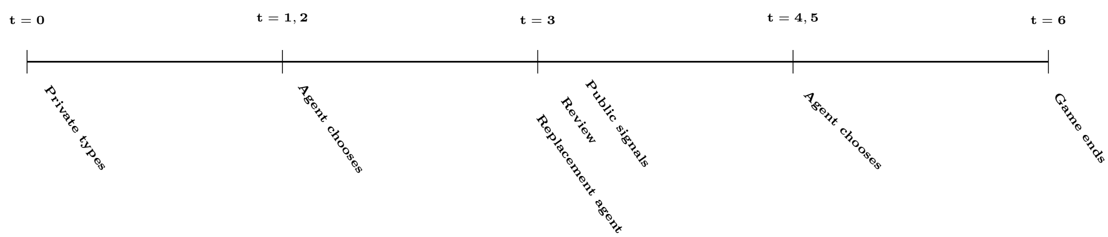


Figure 2.1: Timeline of game with intermittent review

2.3.2 Timing of reviews - intermittent and frequent

Under intermittent review the timing of the game is as follows. Nature moves and picks the type of the agent. The agent serves in office for 2 periods before the review and picks an action from the set $\mathcal{A}$ in *each* period. The principal does not observe the action chosen; instead all[3] players observe a signal about the actions *after both the actions have been chosen*. This feature of the model is meant to capture the fact that the review is intermittent rather than in every period. Review takes place. The principal either keeps (k) or fires (f) the agent based on the signal observed. If the agent is retained, he picks two more actions, each from set $\mathcal{A}$, everyone gets their payoff and the game ends. If the agent is fired then the replacement agent joins the firm. The replacement agent picks two actions and the game ends. The fired agent gets his full utility from pre-review actions and his outside option τ_i in each post review period. The timing is shown in Figure 2.1

In order to meaningfully study the impact of reviewing more often, I construct a comparable game with frequent reviews. The sequence of events is as follows. Nature moves and picks the type of the agent. Whenever the agent is called upon to move, he

[3]The other extreme of this assumption is that employees are aware of the signal their first period action generates at the time that they make the second period decision. Arguably, in most settings neither extreme holds. Instead, employees learn how they did in the first period but their perception need not coincide with the firm's perception of their performance. The assumption I make in this chapter allows me to draw a starker comparison between frequent and intermittent reviews.

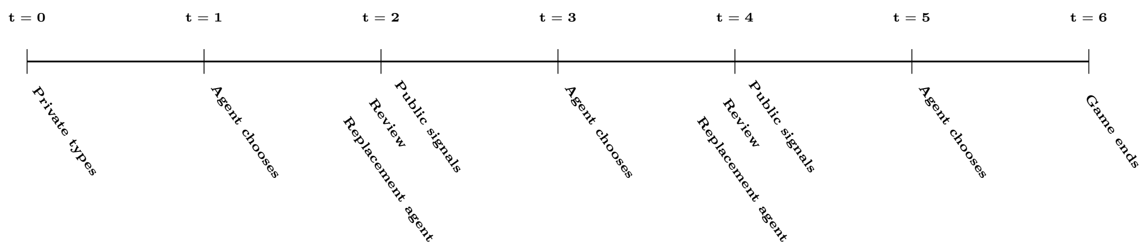


Figure 2.2: Timeline of game with frequent review

chooses an action from the set $\mathcal{A}$. The agent chooses an action. The principal never observes the action chosen, instead all players observe an imperfect signal about the action. Review takes place. The principal either keeps (k) or fires (f) the agent based on the signal observed. If the agent is retained, he picks another action, followed by a public signal regarding his second action and the second review. If the agent is retained once again, he picks one final action and the game ends. If the agent is fired at the first review, then nature picks a "replacement" agent who picks an action, there is a public signal about his action, a review and if retained, he makes another decision before the game ends. If the agent is fired following the second review then the replacement agent makes only one decision. The fired agent gets his full utility from pre-review actions and his outside option τ_i in each post review period. The timing is shown in Figure 2.2.

2.3.3 Equilibria in model with intermittent reviews

In dynamic games with incomplete information, as the one studied here, it is possible that some information sets are not reached with strictly positive probability. If some information sets are not reached with strictly positive probability, Bayes' rule is not applicable at those information sets and it is not clear how the players will assign beliefs to different nodes at these information sets and thus pick one strategy over another. Assumption (i) ensures that under intermittent review no matter which strategy each type of the agent chooses, every information set is reached with strictly positive probability. As a result, several well known equilibrium concepts used in dynamic games of incomplete information - perfect equilibrium, sequential equilibrium, perfect bayesian equilibrium - all yield the same equilibrium outcomes. The same is not true in the game with frequent reviews - all information sets are not reached with strictly positive probability. Following Rogoff and Sibert (1988) and others working on similar problems, I look for sequential equilibria (Kreps and Wilson (1982)). I restrict my attention to pure strategies. We know from Kreps and Wilson (1982) that an equilibrium in mixed strategies always exists. The same is not true when players use pure strategies at every information set, in that it is possible that for some parameter values there is no equilibrium.

Since the game ends two periods after the review, there is no incentive for the agent to choose anything except his preferred action after the review[4]. Knowing this I limit the subsequent analysis to everything that happens up until (and including) the review, taking post review actions in equilibrium as fixed.

The agent makes decisions twice before the review (and then twice after, if he is in office). However, these decisions are made back to back, before any signals arrive or the principal moves, thus they can be rolled into one decision. With that in mind, let the agent's pre-review strategy (henceforth strategy) be given by $b_a = (b_x, b_y)$, where $b_x, b_y \in \hat{\mathcal{A}} \equiv \{XX, XY, YX, YY\}$. b_x and b_y denote the actions chosen by the type x and type y agent respectively. Further if the agent of type i picks actions $b_i \in \hat{\mathcal{A}}$, denote by $q(b_i) = (q_{S_XS_X}, q_{S_XS_Y}, q_{S_YS_X}, q_{S_YS_Y})$ the probability of observing signals $\hat{S} \equiv \{S_XS_X, S_XS_Y, S_YS_X, S_YS_Y\}$ respectively. Under assumption (i) $q(b_i) > 0$ for

[4]If type x is re hired, he picks X in both post review periods. If type y is re hired, he picks Y in both post review periods. If the agent is fired, then with probability α nature selects a type x agent who picks X for two subsequent periods and with probability $1 - \alpha$ nature selects a type y agent who picks Y for the two subsequent periods.

all $b_i \in \hat{\mathcal{A}}$. For example, if the agent chooses $b_a = (XX, XX)$, then the principal observes signals $\{S_X S_X, S_X S_Y, S_Y S_X, S_Y S_Y\}$ with probability $\theta_X^1 \theta_X^2, \theta_X^1(1-\theta_X^2), (1-\theta_X^1)\theta_X^2$ and $(1-\theta_X^1)(1-\theta_X^2)$ respectively, all of which are strictly positive.

In equilibrium the principal's prior on the distribution of types coincides with the actual distribution. Denote by $\gamma^0 = (\gamma_x^0, \gamma_y^0) = (\alpha, 1-\alpha)$, the principal's prior beliefs on the distribution of agent types in equilibrium. There are 4 information sets for the principal, each corresponding to a pair of signals in $\hat{S}$. Each of these information sets has 8 nodes in it. For example the information set where the principal observes the signal $S_X S_X$ could be reached by X or Y playing any of the 4 actions in $\hat{\mathcal{A}}$ available to them. The principal's strategy specifies what he does at each of the 4 information sets described above and is denoted by $b_p = (b_{S_X S_X}, b_{S_X S_Y}, b_{S_Y S_X}, b_{S_Y S_Y})$, where $b_{s_1 s_2} \in \mathcal{P} \equiv \{k, f\}$ for all $s_1 s_2 \in \hat{S}$. Let $\hat{\mathcal{P}} = \{k, f\}^4$. Due to assumption (i) we know that for every action of the agent there is one signal that is generated with the highest probability. With slight abuse of notation, I denote by $b_{b_i} \in \{k, f\}$ the principal's action at the information set generated with highest probability by action $b_i \in \hat{\mathcal{A}}$. So, if $b_i = XX$, then the signal generated with the highest probability is $S_X S_X$. Then b_{XX} denotes the principal's action on observing the signal $S_X S_X$.

Denote by $Prob(keep, b_i, b_p)$ the total probability of agent i being kept on by the principal after the review given b_i and b_p. This probability does not depend on what the other type of agent chooses. Denote by $BR_p(b_x, b_y)$ and $BR_i(b_p)$, where $i \in \{X, Y\}$, the set of actions that are the best response of the principal and the agent of type i respectively. Further, let $\hat{u}_i(b_i)$ be the pre-review utility that accrues to the agent if he chooses $b_i \in \hat{\mathcal{A}}$. For example, if $b_i = XX$, then $\hat{u}_i(b_i) = u_i(X) + u_i(X)$.

Upon observing signal $s_1 s_2 \in \hat{S}$, the principal updates his prior according to the Bayes' rule, which he is able to do at every information set since every information set of the principal is reached with strictly positive probability no matter what the agent chooses. As a consequence, consistent beliefs Kreps and Wilson (1982) can be derived using Bayes' rule at each of the principal's information sets for all b_a. Let the posterior be denoted by $\gamma^r = (\gamma^r_x, \gamma^r_y)$, where the r in the superscript stands for "at the time of review". In what follows I look for equilibrium assessments (strategy-belief pair) however, since beliefs are implied in a straightforward way by the strategies, refer to them as equilibrium strategies.

I refer to actions in which the agent of either type chooses the same action, i.e. $(b_x = b_y)$, as *pooling* actions. For actions in which the two types of the agent use different actions $(b_x \neq b_y)$ I use the term *separating* actions. If both types of agents pick their preferred action in both periods $(b_x = XX, b_y = YY)$, I refer to the action as a *true to type* separating (t-t-t) action.

Equilibria in which the agent chooses pooling, separating, t-t-t actions are called pooling, separating, t-t-t equilibria respectively.

The rest of the analysis proceeds as follows. The principal has 16 (2^4) strategies available to him. The first lemma reduces the strategies of the principal that need to be considered in the subsequent discussion. Lemma 2.2 characterizes the principal's best response function. Then I discuss the agent's incentive to deviate, and how it can be thought of as the sum of two components - the change in pre review utility and the change in probability of being retained after the review. Finally, I present three propositions that characterize the equilibria.

Lemma 2.1. *For all $\tau_i \in [0, u_i(i)]$, $i = \{1, 2\}$ there is no equilibrium in which the principal chooses $b_p \in \{(k,k,k,k),(f,f,f,f)\}$.*

Proof. I consider two classes of strategies that the agent might use and argue that in each of these cases (k,k,k,k) or (f,f,f,f) can never be used in an equilibrium.

Suppose the two types of the agent choose different actions in either of the periods. The principal's posterior assigns the highest probability to facing type x and type y to distinct information sets. He, then, fires the agent at the information set where he believes he is facing the type y agent with the highest probability and keep the agent at the information set where he believes he is facing the type y with the highest probability. Neither (k, k, k, k) and (f, f, f, f) will be used.

Suppose instead, the two types of the agent choose the same actions in both periods. Then principal's prior and posterior are identical and he is indifferent between keeping and firing the agent at each information set. Now further, suppose he does choose to play (k, k, k, k) or (f, f, f, f). At least one of the types of the agent must then be choosing at least one action that is not their most preferred action. Deviating to choosing their most preferred action does not change the probability of being fired and makes that type of the agent strictly better off. So, (k, k, k, k) or (f, f, f, f) cannot arise in this kind of equilibrium either.

□

Lemma 2.1 is going to be implicitly used in all the subsequent analysis. $kkkk$ and $ffff$ are the only two strategies of the principal under which the agent can unilaterally deviate to any action without changing the probability of being retained in his current position. Removing these strategies from those that need to be considered is crucial in pinning down the incentives of the agent to deviate profitably from a strategy profile.

The second lemma identifies the structure of the principal's best response correspondence for pooling and separating actions.

Lemma 2.2. *Let $b_a = (b_x, b_y)$. If $b_x \neq b_y$ then $b_p \in BR_p(b_x, b_y)$ is such that $b_{b_x} = k$, $b_{b_y} = f$. If instead $b_x = b_y$ then $BR_p(b, b) = \hat{\mathcal{P}} \setminus \{(k, k, k, k), (f, f, f, f)\}$.*

Proof. Suppose the agent uses the strategy $b_a = (b_x, b_y)$ and the principal observes the signal $s_1 s_2$ and updates his posterior belief on the distribution of types before the review. Using Bayes' rule, we get:

$$\gamma_x^r = \frac{\alpha q_{s_1 s_2}(b_x)}{\alpha q_{s_1 s_2}(b_x) + (1-\alpha) q_{s_1 s_2}(b_y)}$$

and

$$\gamma_y^r = \frac{(1-\alpha) q_{s_1 s_2}(b_y)}{\alpha q_{s_1 s_2}(b_x) + (1-\alpha) q_{s_1 s_2}(b_y)}$$

γ^r is well defined since $q_{s_1 s_2}(b_x), q_{s_1 s_2}(b_y) > 0$ for all b_x, b_y. The principal keeps the agent he is facing at that information set if

$$2\gamma_x^r v(X) + 2\gamma_y^r v(Y) \geq 2\alpha v(X) + 2(1-\alpha) v(Y)$$

the above inequality can be rearranged and simplified to yield

$$\frac{\alpha(1-\alpha)\{q_{s_1 s_2}(b_x) - q_{s_1 s_2}(b_y)\}\{v(X) - v(Y)\}}{\alpha q_{s_1 s_2}(b_x) + (1-\alpha) q_{s_1 s_2}(b_y)} \geq 0$$

Since $v(X) > v(Y)$ the above inequality holds if and only if

$$q_{s_1 s_2}(b_x) \geq q_{s_1 s_2}(b_y) \tag{2.1}$$

Thus at any information set, the principal keeps the agent he is observing if $q_{s_1 s_2}(b_x) \geq q_{s_1 s_2}(b_y)$, fires the agent he is facing if $q_{s_1 s_2}(b_x) \leq q_{s_1 s_2}(b_y)$ and is indifferent between keeping and firing if $q_{s_1 s_2}(b_x) = q_{s_1 s_2}(b_y)$. Since at every $s_1 s_2$, $q_{s_1 s_2}(b_x) \geq q_{s_1 s_2}(b_y)$ or $q_{s_1 s_2}(b_x) \leq q_{s_1 s_2}(b_y)$ and thus the principal's best response set is non empty.

Further, when $b_x = b_y$, that is the agent chooses a pooling action, then $q_{s_1 s_2}(b_x) = q_{s_1 s_2}(b_y)$ and the principal is indifferent between keeping and firing the agent. In this case $BR_p(b, b) = B_p \setminus \{(k, k, k, k), (f, f, f, f)\}$. When $b_x \neq b_y$, then $b_p(b_x) = k$ and $b_p(b_y) = f$. $\square$

Next, I study the structure of the best response correspondence of the agent.

Suppose the principal uses strategy $b_p \in \hat{P} \setminus \{(k, k, k, k), (f, f, f, f)\}$ and the type $i, i \in \{X, Y\}$ agent chooses $b_i \in \hat{\mathcal{A}}$, he gets:

$$U(b_i; b_p) = \hat{u}_i(b_i) + 2Prob(keep, b_i, b_p)u_i(i) + 2(1 - Prob(keep, b_i, b_p))\tau_i$$

If instead, he unilaterally deviates to $\tilde{b}_i \in \hat{\mathcal{A}}$, he gets:

$$U(\tilde{b}_i; b_p) = \hat{u}_i(\tilde{b}_i) + 2Prob(keep, \tilde{b}_i, b_p)u_i(i) + 2(1 - Prob(keep, \tilde{b}_i, b_p))\tau_i$$

This deviation is profitable if and only if:

$$\hat{u}_i(\tilde{b}_i) - \hat{u}_i(b_i) + 2(Prob(keep, \tilde{b}_i, b_p) - Prob(keep, b_i, b_p))(u_i(i) - \tau_i) > 0 \quad (2.2)$$

Since $\tau_i \leq u_i(i)$ we have that $u_i(i) - \tau_i \geq 0$. Thus the gains from deviation can be broken down into two components:

1. $\hat{u}_i(\tilde{b}_i) - \hat{u}_i(b_i)$: the gain in pre-review utility.
2. $(Prob(keep, \tilde{b}_i, b_p) - Prob(keep, b_i, b_p))(u_i(i) - \tau_i)$: the gain from the probability of being retained following the review.

The following propositions characterize the set of equilibria. I start by showing that there no separating equilibria that are not t-t-t. Next, I show that separating equilibria exist for higher values of outside options and pooling equilibria *may* exist for lower values.

Proposition 2.1. *Every separating equilibrium is t-t-t.*

Proof. Consider a separating action that is not t-t-t, and pick any $b_p \in BR_p(b_x, b_y)$. I will show that such an action can never arise in equilibrium.

Since b_a is a separating action that is not t-t-t, there is at least one agent, say agent i, and a strategy $\tilde{b}_i \in \hat{\mathcal{A}}$ such that $\hat{u}_i(b_i) < \hat{u}_i(\tilde{b}_i)$. Further, for any $\tilde{b}_i \in \hat{\mathcal{A}} \setminus \{b_i\}$, $Prob(keep, b_i, b_p) - Prob(keep, \tilde{b}_i, b_p) \leq 0$. Then from (2.2) we know that agent i can unilaterally deviate to $\tilde{b}_i$ and do strictly better. Thus b_a can't arise in equilibrium.

□

The intuition behind the result is simple. When the two types of the agent use different actions the principal is able to tell which agent is most likely facing. He then proceeds to fire the type y agent and keep the type x agent, irrespective of which actions they chose. As a result there is no incentive for the agent to choose anything other than his most preferred action. Thus any separating equilibrium that might exist must be t-t-t.

In a true to type separating equilibrium $b_x = XX$ and $b_y = YY$. From lemma 2.2 we know that the principal will keep on observing signal $S_X S_X$ and fire on observing $S_Y S_Y$. The principal's best response on observing the other two information sets depends on the value of the θ's, that is $BR_p(XX, YY) \in \{kkkf, kkff, kfkf, kfff\}$.

When the agent uses t-t-t actions, neither of the types of the agent can deviate and improve their pre-review utility for any b_p. Further, for any $b_p \in BR_p(XX, YY)$, the type X agent can't deviate to any other strategy and increase the probability of being retained. Thus the type x agent has no incentive to deviate. This also implies that the existence of a t-t-t equilibrium does not depend on τ_x. The same is not true for the type y agent. While it continues to be true that he can't deviate and increase his pre-review payoff; he can increase his probability of being retained. The key trade-off here is the loss of pre-review utility versus the gain in the likelihood of retention. If the gain in probability of retention does not compensate the agent for the loss of pre-review utility then the t-t-t action is an equilibrium. Further, from inequality (2.2) note that as τ_y increases, the left hand side decreases. The following proposition follows directly from inequality (2.2).

Proposition 2.2. *There exists $0 \leq \tau^{I*} < u_Y(Y)$, such that there exists a t-t-t equilibrium for all $\tau \geq \tau^{I*}$ and there is no t-t-t equilibrium for $\tau < \tau^{I*}$*

To summarize, the only separating equilibria that can exist in this model are t-t-t separating equilibria. For a given set of parameters, a t-t-t equilibrium is more likely to exist for high values of τ_y.

Next, I turn my attention to pooling equilibria. When both types of the agent pool by choosing the same action in both periods, the principal's posterior at every information set coincides with his prior as shown in lemma 2.2. The principal is indifferent between keeping and firing the agent at every information set. Thus, unlike separating actions, the principal's best response function can't be used to narrow down the set of profiles that may arise in pooling equilibria.

Pooling equilibria may arise at any of the actions of the set $\hat{\mathcal{A}}$ and in every pooling action there is at least one agent who can unilaterally deviate and increase his pre-review utility. From inequality (2.2) it is clear that for any of these pooling actions to arise in equilibrium it must be that the probability of being retained strictly decreases from deviating to any other strategy. Not only that, it must decrease enough to make the gain in pre-review utility insufficient to compensate for reduction in retention probability. This fact makes it possible to narrow down the candidates for pooling equilibria. The strategy profiles that can be pooling equilibria are given in appendix 1.

As discussed above the crucial incentive that makes it possible for pooling equilibria to exist in this model is the increased probability of being fired by deviating. From inequality (2.2) it can be seen that as τ_x and τ_y increase, the second term of the left hand side decreases. The decrease in retention rate becomes less important. In fact, as the following proposition shows, if a pooling equilibrium exists for $\tau_i = 0$, then as τ_i increases the pooling equilibrium continues to exist until some cutoff τ^{I**} after which it no longer exists.

Proposition 2.3. *If there is a pooling equilibrium for $\tau_i = 0$*[5] *then there exists a $0 < \tau_i^{I**} < Eu_i$, where $Eu_i = \alpha u_i(X) + (1-\alpha)u_i(Y)$, such that there is pooling for every $\tau_i \leq \tau_i^{I**}$ and no pooling for $\tau_i^{I**} < \tau_i$.*

Proof. Consider the pooling equilibrium in which both types of agents choose XX. From lemma 2 we know that the principal is indifferent between keeping and firing at every information set. We also know from observation 4 there are only 3 strategies of the principal that can arise in equilibrium - $(k,k,k,f),(k,f,f,k),(k,f,f,f)$. In either of these strategy profiles the type x agent has no incentive to deviate and the existence of the equilibrium does not depend upon τ_x. That said, for these profiles to be equilibria the type y agent must not have an incentive to unilaterally deviate. In order for Y to not have the incentive to deviate from XX to $\tilde{b}_Y \in \{XY, YX, YY\}$ the following condition needs to hold:

$$2u_Y(X) - \hat{u}_Y(\tilde{b}_Y) + 2\{Prob(keep, XX, b_p) - Prob(keep, \tilde{b}_Y, b_p)\}\{u_Y(Y) - \tau_y\} \geq 0$$

[5] The existence of pooling at XX depends only on τ_y, that of pooling at XY and YX depends on both τ_x and τ_y and of pooling at YY depends only on τ_x. Given this, is there a better way to state this theorem?

If the inequality holds for all $\tilde{b}_Y$ for a value of τ_y and b_p then (XX, XX, b_p) is a pooling equilibrium. Notice that $2u_Y(X) - \hat{u}_Y(\tilde{b}_Y) < 0$. If the above condition holds, it must be that $\{Prob(keep, XX, b_p) - Prob(keep, \tilde{b}_Y, b_p)\} > 0$ and the gain in probability of being kept from playing XX overwhelms that loss of pre-review utility.

As τ_y increases the left hand side of this inequality becomes smaller. If the above conditions hold for all $\tilde{b}_Y$ for $\tau_y = 0$, then they also hold for $\tau_y = \epsilon > 0$, where $\epsilon \to 0$. For each $\tilde{b}_Y$, let the value of τ_y that makes left hand side equal to 0 be $\tau_{y,\tilde{b}_Y}$. Then $\tau_{y,\tilde{b}_Y}$ is given by:

$$\tau_{y,\tilde{b}_Y} = \frac{2u_Y(Y)\{Prob(keep, XX, b_p) - Prob(keep, \tilde{b}_Y, b_p)\} - \hat{u}_Y(\tilde{b}_Y) + 2u_Y(X)}{2\{Prob(keep, XX, b_p) - Prob(keep, \tilde{b}_Y, b_p)\}}$$

From the argument made above, notice that $\tau_{y,\tilde{b}_Y} > 0$. Further, we can compare $\tau_{y,\tilde{b}_Y}$ with Eu_Y. With a little algebra it can be shown that if $\tilde{b}_Y = \{XY, YX\}$ then

$$Eu_Y - \tau_{y,\tilde{b}_Y} = \frac{(1 + 2\alpha\{Prob(keep, XX, b_p) - Prob(keep, \tilde{b}_Y, b_p)\})(u_Y(Y) + u_Y(X))}{\{Prob(keep, XX, b_p) - Prob(keep, \tilde{b}_Y, b_p)\}}$$

If instead $\tilde{b}_Y = \{YY\}$ then

$$\begin{aligned} Eu_Y - \tau_{y,\tilde{b}_Y} = \\ &\frac{2\alpha\{Prob(keep, XX, b_p) - Prob(keep, \tilde{b}_Y, b_p)\})u_Y(X)}{\{Prob(keep, XX, b_p) - Prob(keep, \tilde{b}_Y, b_p)\}} \\ &+ \frac{2(1-\alpha)\{Prob(keep, XX, b_p) - Prob(keep, \tilde{b}_Y, b_p)\}u_Y(Y)}{\{Prob(keep, XX, b_p) - Prob(keep, \tilde{b}_Y, b_p)\}} \end{aligned}$$

In either case $Eu_Y - \tau_{y,\tilde{b}_Y} > 0$ and so $Eu_Y > \tau_{y,\tilde{b}_Y}$.

Set $\tau_y^{**} = min_{\tilde{b}_Y} \tau_{y,\tilde{b}_Y}$. Clearly, $0 < \tau_y^{**} < Eu_Y$. Further, if XX was a pooling equilibrium for $\tau_y = 0$, then it continues to be a pooling equilibrium for $0 < \tau_y < \tau_y^{**}$.

Analogous arguments can be used to show that the same result applied for pooling equilibria at XY, YX, YY.

□

Thus, when the outside option of the agent is low then review is likely to fail in its primary objective of weeding out employees who are a bad match for the firm. The ill-matched employee has the incentive to blend in with the well-matched employee and try to game the review process. That said, as the outside options increase, this stops being the case and the ill-matched employees reveal themselves during the review.

Example 2.1. Suppose $\tau_x = \tau_y = 0$ and the rest of the parameters have the following values:

$$\theta_X^1 = 0.9 \qquad \theta_X^2 = 0.8$$

$$\theta_Y^1 = 0.7 \qquad \theta_Y^2 = 0.6$$

$$u_X(X) = 4 \qquad u_X(Y) = 2$$

$$u_Y(X) = 2 \qquad u_Y(Y) = 3$$

Then $(XX, XX, kfff)$, $(XY, XY, fkff)$ and $(YX, YX, ffkf)$ are (pooling) equilibria. On setting $u_X(X) = 3$ while keeping all other parameter values the same, $(YY, YY, fffk)$ is also an equilibrium in addition to the above four.

The values of τ_x and τ_y can be increased all the way to 1.2 and all of the pooling equilibria that existed for $\tau_i = 0$ continue to exist. For very low values of τ_y the model doesn't have t-t-t separating equilibria but for all $\tau_y > .6$, the model has a t-t-t separating equilibrium. In fact, for $.6 \leq \tau_y \leq 1.2$, the model has both pooling and t-t-t equilibria. The model displays multiplicity of equilibria for several parameter values.

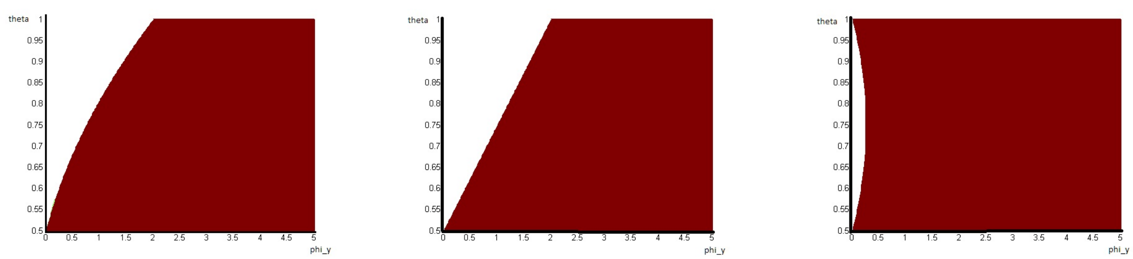
It is worth noting that inequality (2.2) can be rearranged for all $b_i, \tilde{b}_i$ and b_p such that the incentive of the agent to deviate depends on the ratio of the cost of choosing one's preferred action, $u_i(i) - u_i(j)$, and the cost of being fired, $u_i(i) - \tau_i$. Let

$$\frac{u_X(X) - u_X(Y)}{u_X(X) - \tau_x} = \phi_X \text{ and } \frac{u_Y(Y) - u_Y(X)}{u_Y(Y) - \tau_y} = \phi_Y$$

Thus, inequality (2.2) can be rewritten in terms of $(Prob(keep, \tilde{b}_i, b_p) - Prob(keep, b_i, b_p))$, ϕ_X and ϕ_Y.

The example that follows illustrates the pooling and separating equilibria for a special case.

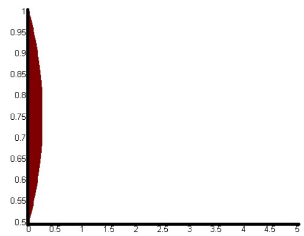
Example 2.2. Set $\theta_X^1 = \theta_X^2 = \theta_Y^1 = \theta_Y^2 = \theta$ and $\phi_X = \phi_Y = \phi$. In the following figures the shaded region shows the values of θ and ϕ for which different equilibria exist in this model. On the X-axis I show values of ϕ and on the Y-axis values of θ. Figure 2.3 shows the region where t-t-t equilibria exist and Figure 2.4-Figure 2.6 show the regions where the pooling equilbria exist.[6] The first thing to note from Figure 2.3-Figure 2.6 is that in this case an equilibrium always exists. As mentioned earlier, this is not true for the general case. Secondly, as in example 2.1, there is multiplicity, in that there are parameter values for which there is more than one equilibrium.



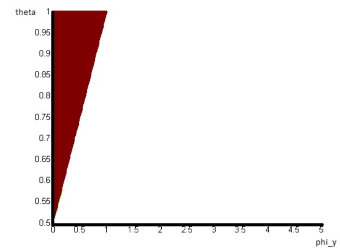
(a) Equilibrium I (b) Equilibrium II-III (c) Equilibrium IV

Figure 2.3: Parameters for which t-t-t equilibria exist

[6] Equilibrium I $= (XX, YY, kkkf)$, Equilibrium II$=(XX, YY, kkff)$, Equilibrium III$=(XX, YY, kfkf)$, Equilibrium IV$=(XX, YY, kfff)$, Equilibrium 1a-1c = pooling equilibria with $b_x = b_y = XX$, Equilibrium 2a-2c=pooling equilibria with $b_x = b_y = XY$, Equilibrium 3a-3c=pooling equilibria with $b_x = b_y = YX$, Equilibrium 4a-4c=pooling equilibria with $b_x = b_y = YY$

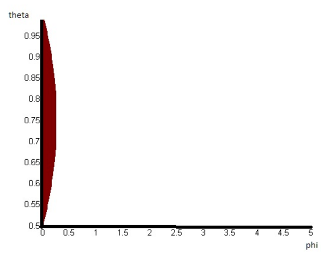


(a) Equilibrium 1.a

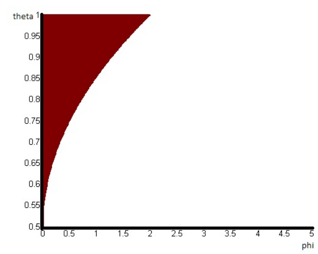


(b) Equilibrium 1.c

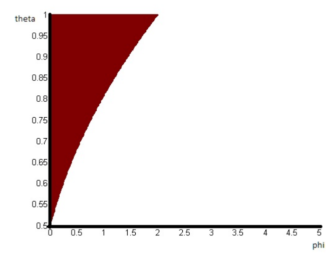
Figure 2.4: Parameters for which pooling equilibria at XX exist



(a) Equilibrium 2.a-3.a

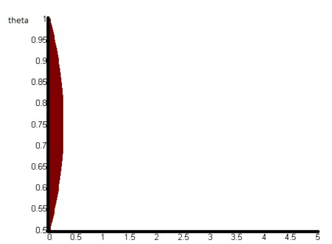


(b) Equilibrium 2.b-3.b

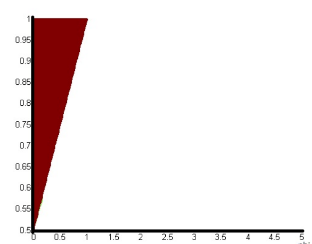


(c) Equilibrium 2.c-3.c

Figure 2.5: Parameters for which pooling equilibria at XY and YX exist



(a) Equilibrium 4.a



(b) Equilibrium 4.c

Figure 2.6: Parameters for which pooling equilibria at YY exist

Proposition 2.2 and 2.3 showed that increasing the value of τ makes is easier for the conditions for separating equilibria to hold and harder for conditions for pooling equilibria to hold. Increasing τ is equivalent to increasing ϕ. These results can be clearly seen in these figures as well.

2.3.4 Equilibria in model with frequent reviews

In this section, I analyze the model where review occurs more frequently (twice, once after every action). I compare the equilibrium outcomes under the two review regimes (intermittent and frequent). I examine t-t-t and pooling equilibria and argue that analogous results to proposition 2.2 and 2.3 continue to hold in this model. Further, I compare thresholds at which pooling stops existing under the two review regimes in the special case where $\theta_X^1 = \theta_X^2 = \theta_X$ and $\theta_Y^1 = \theta_Y^2 = \theta_Y$. The efficacy of the review depends on how θ_X and θ_Y compare and on the order of the pooling actions. In particular, I show that whether or not frequent reviews are better than intermittent review depends on several factors.

As before, a true to type (t-t-t) action is one in which the agent chooses his preferred action every time it is his turn to move. A pooling action is one in which both types of the agent choose identical actions in each period. For example, a pooling action at XY is when in the first period both types of the agent choose X and in the second period they both choose Y at every information set. Finally, in this model there exist separating equilibria that are not t-t-t.

Once again, at the very last decision making period, whichever agent is in office picks his favorite policy in equilibrium and thus, I treat that as fixed and only consider decisions up to and including the second review. Further, if the agent is fired after the first review, then the game that follows is as shown in Figure 2.7. I discuss the subgame in detail for two reasons. First, because it is a subgame of the current game and the outcomes are relevant to the subsequent analysis. Secondly, because it is useful in understanding the results in the model with intermittent review.

There are four strategies available to the agent - $\{(X,X),(X,Y),(Y,X),(Y,Y)\}$. The principal has two information sets. At the first information set, he observes the signal S_X and at the second information set he observes the signal S_Y. Thus, the principal's strategies can be one of the following - $\{kk, kf, fk, ff\}$. Since all

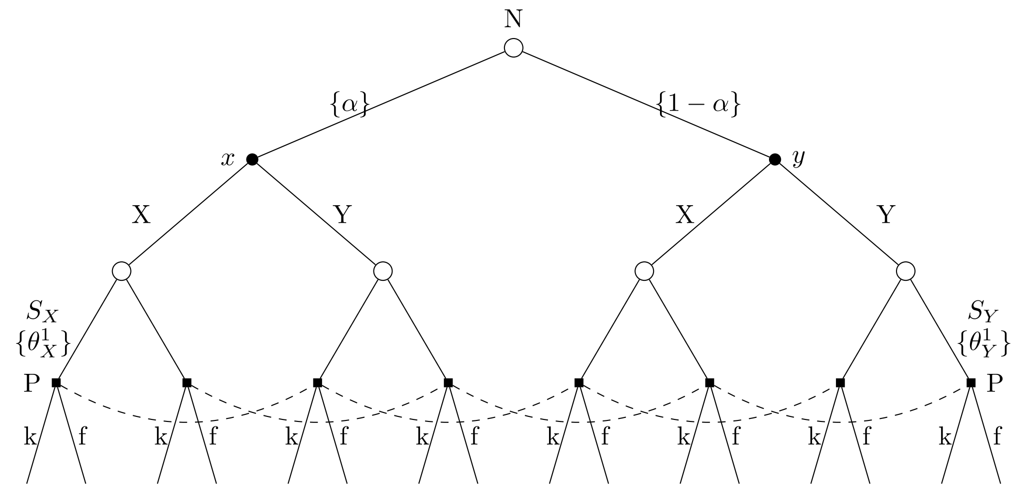


Figure 2.7: Game tree for model with one decision, one review

information sets are reached with strictly positive probability, calculating consistent beliefs associated with every strategy using Bayes' rule is straightforward.

It can be shown that in equilibrium, the principal's strategies $\{kk, ff\}$ are never used[7]. Further, it can be shown that the best response of the principal to the agent choosing (X, Y) is kf and to (Y, X), it is fk [8]. The agent's incentive to deviate can be broken down into gains in pre review utility and gain in the probability of being retained.

(Y, X, fk) is not an equilibrium because the agent of type y can unilaterally deviate to choosing Y and gain pre review utility as well as increase the probability of being kept. Thus there is no separating equilibrium that is not true to type. In the model with two decisions before the review, the same is true and is stated as proposition 2.1.

[7] See the Lemma 1, which shows the 2 decision analogue of this claim.

[8] See Lemma 2

Strategy profile (X, Y, kf) is the t-t-t separating action profile and the only candidate to be a t-t-t equilibrium. Notice that, neither of the types of the agent can gain pre review utility by deviating. In addition, the type x agent can't deviate and increase the probability of being retained. As a result the type x agent has no incentive to deviate. The type y agent can deviate and choose action X and this will increase his probability of being retained. However, as long as the following condition is satisfied, the agent will not have the incentive to deviate and (X, Y, kf) is an equilibrium.

$$\frac{u_y(Y) - u_y(X)}{u_y(Y) - \tau_y} \geq \theta_X^2 + \theta_Y^2 - 1 \tag{2.3}$$

When the agent chooses (X, X) or (Y, Y), the principal is indifferent between keeping and firing the agent at every information set because his posterior on agent distribution at the time of review is the same as his prior. If, however, he uses the strategy fk in response to (X, X) the the agent of type y would unilaterally deviate to choose Y. He would not only would gain pre review utility but would also increase the probability of being retained. Thus the only possible strategy profile in which the agent chooses (X, X) that could arise in equilibrium is (X, X, kf). The type x agent has no incentive to deviate. The type y agent can gain pre review utility by deviating but will also lose some retention probability. The profile is an equilibrium as long as the following condition holds:

$$\frac{u_y(Y) - u_y(X)}{u_y(Y) - \tau_y} \leq \theta_X^2 + \theta_Y^2 - 1 \tag{2.4}$$

By a similar argument it can be shown that the only possible strategy profile in which the agent chooses (Y, Y) that could arise in equilibrium is (Y, Y, fk). The condition that needs to hold in order for this profile to be an equilibrium is

$$\frac{u_x(X) - u_x(Y)}{u_x(X) - \tau_x} \leq \theta_X^2 + \theta_Y^2 - 1 \tag{2.5}$$

Inequalities (2.3)-(2.5) can be used to infer the nature of t-t-t and pooling equilibria in this model. The first thing to note is that as τ_y increases the left hand side of inequalities (2.3) and (2.4) increases. In fact, when $\tau_y = u_y(Y)$ then inequality (2.3) holds strictly and there is a t-t-t separating equilibrium and inequality (2.4) fails to hold so there is no pooling at (X, X). Thus t-t-t equilibria are more likely to exist for higher values of τ_y and pooling equilibria at lower values. This is exactly the content of propositions 2.2 and 2.3.

Further, notice that for all possible parameter values either inequality (2.3) or (2.4) holds. Thus an equilibrium always exists. While all the preceding conclusions from the one-decision model turn out to hold for the model with two decisions, this one does not. That is to say, that in the two decision case, an equilibrium does not exist for all parameter values.

For any t-t-t separating and pooling action to be an equilibrium the above subgame must have one of the equilibria specified above. For example, in a t-t-t equilibrium it must be that in the above subgame the equilibrium is (X, Y, kf). Similarly, in a pooling equilibrium where agents pool by playing X every time they are called upon to move, the equilibrium of the above subgame must be (X, X, kf) and so on. Since we know the conditions that need to hold in that subgame, I define the strategies of all players restricted to the portion of the tree where the agent is retained at the time of the first review for the rest of the analysis.

Let the agent's strategy be denoted by $c_a = (c_x, c_y) \in \mathcal{A}^6$, where $c_i \in \mathcal{A}^3$ specifies what the type i agent does before the first review and then after the first review if he is retained and the first period signals were S_X and S_Y respectively. For example, if $c_x = (X, X, Y)$ then the agent of type x pick X before the first review, if he is retained then if the signal before the review was S_X he picks X again and if instead, the signal was S_Y he picks Y. The principal's strategy specifies what he does at the first review upon observing each signal, and what he does after observing each signal *if* he retained the agent in the first review. Of course, the principal also makes a decision at the second review if he had fired the agent in the first review, but as discussed earlier, we know what his best response will be in different equilibria that can possibly arise in that subgame. Thus, we can think of the principal' strategy as $d_p \in \mathcal{P}^6$, which specifies what the principal does on observing $\{S_X, S_Y\}$, and if he retains the agent, $\{S_X S_X, S_X S_Y, S_Y S_X, S_Y S_Y\}$.

The fact that I only consider t-t-t and pooling equilibria simplifies the analysis considerably because it imposes the condition that the agent, irrespective of type, chooses the same action at every information set if retained in the first review. As a result, the agent's strategy can be thought of as analogous to those in subsection 3.3, in that each type of agent makes one decision in the first period (before the review) and then one decision in the second period (after the review). Denote by $d_a = (d_x, d_y) = (d_{x1} d_{x2}, d_{y1} d_{y2})$ the agent's (t-t-t or pooling) strategy, where d_{it} is the type i agent's action at time t. For instance, a t-t-t strategy of the agent is (XX, YY).

One of the big differences between this model and the model with intermittent review is that not all information sets are reached with strictly positive probability. In order to find consistent beliefs Kreps and Wilson (1982) for a strategy profile (d_a, d_p), I construct a purely mixed strategy $(d_{a,\epsilon}, d_{p,\epsilon})$ which assigns probability $1 - \epsilon$ to the action prescribed by (d_a, d_p) at every information set and ϵ to the *other* action. Since the action set at every information set of every player is binary, this is can be done for any (d_a, d_p). For this purely mixed strategy profile, it is possible to construct beliefs μ_ϵ at every information set using Bayes' rule. The limit of $(d_{a,\epsilon}, d_{p,\epsilon}, \mu_\epsilon)$ needs to be sequentially rational to be an equilibrium assessment. As ϵ goes to 0, $(d_{a,\epsilon}, d_{p,\epsilon})$ converges to (d_a, d_p) by construction and μ_ϵ converges to μ. The agent's information sets are always singletons and μ accordingly sets probability 1 to each of these. Both of the principal's information sets at the time of the first review are reached with strictly positive probability and μ derives beliefs using the Bayes' rule. Some of the principal's information sets following the first review might not be reached, but μ being the limit of a purely mixed assessment is able to assign well defined beliefs to the nodes at each of these information set. Let $q_{s_1}(d_{i1})$ be the probability of observing signal s_1 given that agent i chooses action d_{i1}. $q_{s_1 s_2}(d_{i1}d_{i2})$ denotes the probability of observing signal $s_1 s_2$ if the agent of type i chooses $d_{i1}d_{i2}$ *if* the principal retains the agent in the first review. μ can be used to construct the posterior beliefs of the agent about the distribution of types at every information set at the time of both reviews. Let $\gamma^{r_1} = (\gamma_x^{r_1}, \gamma_x^{r_1})$ and $\gamma^{r_2} = (\gamma_x^{r_2}, \gamma_x^{r_2})$ be the principal's belief about the distribution of the type of agents at the time of the first and second review respectively. Then on observing signal s_1,

$$\gamma_x^{r_1 1} = \frac{\alpha q_{s_1}(d_{x1})}{\alpha q_{s_1}(d_{x1}) + (1-\alpha) q_{s_1}(d_{y1})} \tag{2.6}$$

And if he retains the agent, then on observing $s_1 s_2$

$$\gamma_x^{r_1 2} = \frac{\alpha q_{s_1 s_2}(d_{x1}d_{x2})}{\alpha q_{s_1 s_2}(d_{x1}d_{x2}) + (1-\alpha) q_{s_1 s_2}(d_{y1}d_{y2})} \tag{2.7}$$

For example, suppose the type x and y agents choose X and Y respectively, every time they are called upon to choose. In the first review, the principal fires on observing S_Y and keeps on observing s_X. If he retained the agent in the first review, then he chooses $kfkf$ on observing signals $S_X S_X, S_X S_Y, S_Y S_X, S_Y S_Y$ respectively.

Thus, $d_a = (XX, YY)$ and $d_{x1} = d_{x2} = X$, while $d_{y1} = d_{y2} = Y$. On observing signal S_X

$$\gamma_x^{r_1} = \frac{\alpha\theta_X^1}{\alpha\theta_X^1 + (1-\alpha)(1-\theta_Y^1)}$$

Further, if he doesn't fire the agent and subsequently observes signal S_Y, then

$$\gamma_x^{r_1} = \frac{\alpha\theta_X^1(1-\theta_X^2)}{\alpha\theta_X^1(1-\theta_X^2) + (1-\alpha)(1-\theta_Y^1)\theta_Y^2}$$

Notice that (2.7) is exactly the same as the posterior belief of the principal given in lemma 2.2 and using the same argument we can characterize the best response of the principal during the second review if he had retained the agent in the first review. If $q_{s_1s_2}(d_{x1}d_{x2}) > (<,=) q_{s_1s_2}(d_{y1}d_{y2})$, then the principal keeps (fires, is indifferent between keeping and firing) the agent. Thus, for t-t-t equilibrium the principal's best response lies in the set $\{kkkf, kkff, kfkf, kfff\}$ and for pooling actions, that principal is indifferent at every information set as in subsection 3.3.

Next, I consider the agent's best response when he is called upon to choose an action between the first and the second review. In order for the agent to be willing to choose d_{i2} rather than deviating to $\tilde{d}_{i2}$, the following condition must hold:

$$u_i(d_{i2}) - u_i(\tilde{d}_{i2}) + (Prob(keep, d_p, d_{i2}) - Prob(keep, d_p, \tilde{d}_{i2}))(u_i(i) - \tau_i) \geq 0 \quad (2.8)$$

In the case of t-t-t actions, the above equation can be used to make an argument identical to the one made in subsection 3.3 as well as earlier in this section. Type x agent has no incentive to deviate. In order for type y not to deviate the gain in retention probability must be less than the loss in pre-review utility of deviation. This yields the following condition which is identical to (2.3)

$$\frac{u_y(Y) - u_y(X)}{u_y(Y) - \tau_y} \geq \theta_X^2 + \theta_Y^2 - 1$$

Inequality (2.8) can be used to identify the strategies of the principal that can arise in equilibrium in the case of pooling actions using the same argument as in subsection 3.3. In addition, it can be used to find the conditions that need to hold for the agent to adhere to the pooling action. Unsurprisingly, these conditions are identical to the ones derived earlier in this section for the subgame with one decision and one review. For pooling at XX, YX we need (2.4) and for pooling at XY, YY we need (2.5).

Next, I consider the decision of the principal at the first review, having observed the first signal. Let $V(X, s_1, d_{x2}, d_{y2}, d_{p2})$ be the principal's expected payoff if the retained agent on observing signal s_1 is a type x agent. This payoff depends on the strategy used by both types of agents and his own strategy in the second period. The principal will keep the agent on observing signal s_1 if

$$\gamma_x^{r_1} V(x, s_1, d_{x2}, d_{y2}, d_{p2}) + (1 - \mu_{s_1}(X)) V(x, s_1, d_{x2}, d_{y2}, d_{p2}) \geq V(\Gamma)$$

where Γ is the equilibrium strategy profile in the subgame that ensues if the agent is fired and $V(\Gamma)$ is the expected payoff to the principal therein. For example, in the analysis of a t-t-t equilibrium, the relevant equilibrium in the subgame following firing is (X, Y, kf) and

$$V(X, Y, kf) = \alpha[v(X) + \theta_X^2 v(X) + (1 - \theta_X^2 Ev] + (1 - \alpha)[v(Y) + \theta_Y^2 v(Y) + (1 - \theta_Y^2) Ev]$$

In the case of t-t-t equilibria, the continuation values received by the principal upon keeping and firing the agent may not be the same depending on the strategy that the principal is using at the second review. However, $V(x, s_1, d_{x2}, d_{y2}, d_{p2}) \geq V(y, s_1, d_{x2}, d_{y2}, d_{p2})$. This coupled with the fact that the γ^{r_1} is more precise than γ^0 implies that the best response of the principal is to keep on observing S_X and fire on observing S_Y.In the case of pooling equilibria, the continuation values that follow on keeping and firing the agent are the same and the principal's prior and posterior are identical implies that the principal is indifferent between keeping and firing the agent at every information set.

Finally, I analyze the decision that the agent makes before the first review. The type i agent can choose d_{i1} as prescribed by the strategy or unilaterally deviate to $\tilde{d_{i1}}$. In doing so, he changes his pre-review utility and the probability of being retained. If the following condition holds, he has no incentive to unilaterally deviate:

$$u_i(d_{i1}) - u_i(\tilde{d_{i1}}) + (Prob(keep, d_p, d_{i1}) - Prob(keep, \tilde{d_{i1}}))(U(d_{i2}, d_p) - 2\tau_i) \geq 0 \quad (2.9)$$

where $U(d_{i2}, d_p)$ is the continuation value that the type i agent receives if he is retained. In the case of a t-t-t equilibrium, $U(d_{i2}, d_p) - 2\tau_i \geq 0$ for both types of agents and neither type of agent can increase his pre review utility by unilaterally deviation. Further, the type x agent can't even increase his probability of being retained by unilaterally deviating. Thus he has no incentive to deviate. The type y agent can increase his probability of being retained by unilaterally deviating and in order to prevent him from doing some conditions need to be hold (given in appendix 2).

In case of pooling equilibrium, there is always at least one agent who can increase his pre-review utility by unilaterally deviating. This gives a guideline for determining what the principal will do at the time of the first review. It has to be true that the probability of being retained must strictly decrease on deviating to the non-pooling action. In addition, some conditions need to hold in order to deter deviation. These conditions are given in appendix 3.

Note from (2.8) and (2.9) that propositions analogous to 2.2 and 2.3 hold in this model. These are given below.

Proposition 2.4. *There exists* $0 \leq \tau^{F*} < u_Y(Y)$*, such that there exists a t-t-t equilibrium for all* $\tau \geq \tau^{F*}$ *and there is no t-t-t equilibrium for* $\tau < \tau^{F*}$

Proposition 2.5. *If there is a pooling equilibrium for* $\tau_i = 0$[9] *then there exists a* $0 < \tau_i^{F**} < Eu_i$*, where* $Eu_i = \alpha u_i(X) + (1-\alpha)u_i(Y)$*, such that there is pooling for every* $\tau_i \leq \tau_i^{F**}$ *and no pooling for* $\tau_i^{F**} < \tau_i$*.*

That is to say, that as τ increases separating equilibria exist and pooling equilibria, if they existed, cease to do so. Thus, irrespective of the review regime low values of τ will lead to a failure to weed out ill matched employees. Even though this is a negative result, it does help understand the conditions under which reviews don't work well irrespective of how frequently they are conducted. Certain careers and industries offer low outside options. In such environments, it might be difficult to design reviews that work.

This proposition allows me to compare the thresholds for pooling equilibria for the model with intermittent review and those in the model with frequent review for the special case when $\theta_X^1 = \theta_X^2 = \theta_X$ and $\theta_Y^1 = \theta_Y^2 = \theta_Y$. Let τ_i^{I**} and τ_i^{F**} be the thresholds at which pooling no longer exists for agent i under intermittent(I) and frequent (F) review respectively. I find that

1. For pooling at XX: If $\theta_X > \theta_Y$ then $\tau_Y^{F**} < \tau_Y^{I**}$ and if $\theta_X < \theta_Y$ then $\tau_Y^{F**} > \tau_Y^{I**}$

[9] The existence of pooling at XX depends only on τ_y, that of pooling at XY and YX depends on both τ_x and τ_y and of pooling at YY depends only on τ_x. Given this, is there a better way to state this theorem?

2. For pooling at YY: If $\theta_X > \theta_Y$ then $\tau_X^{F**} > \tau_X^{I**}$ and if $\theta_X < \theta_Y$ then $\tau_X^{F**} < \tau_X^{I**}$

3. For pooling at XY: $\tau_X^{F**} < \tau_X^{I**}$ and $\tau_Y^{F**} > \tau_Y^{I**}$

4. For pooling at YX: $\tau_X^{F**} > \tau_X^{I**}$ and $\tau_Y^{F**} < \tau_Y^{I**}$

The above inequalities show that the informativeness of the signals matters a great deal.

I conclude the chapter by discussing the second kind of failure in reviews. As shown in proposition 2.1, the model with intermittent reviews has no separating equilibria that are not t-t-t. The same is not true for the model with frequent reviews. In fact, there are several non-t-t-t separating equilibria. In any non-t-t-t separating equilibrium, the probability of weeding out the ill-matched employee is higher than in pooling equilibria but lower than in t-t-t separating equilibria. The reason is that the principal bases his decision on signals generated by only one action rather than two. This allows the ill-matched agent to mimic the well-matched agent in one period, not get fired and then take his preferred action in other periods. In other words, frequent reviews make "toeing the party line" less costly for the ill-matched employees. This kind of failure, while more subtle, arises only with frequent reviewing.

Frequent reviews are by no means prima facie superior to intermittent reviews. In fact, in certain environments intermittent reviews may fare better. If we add to that the fact that more frequent reviews usually cost more, it becomes less obvious that firms want to adopt them indiscriminately.

2.4 Conclusion and discussion

Performance reviews are a ubiquitous feature of employee management within firms and that raises questions about their efficacy and the factors that might enhance or diminish it. This paper shows that the outside options that an employee faces on leaving the firm can have a significant impact on the outcomes of the review process. In particular, if the firm wants to use the annual reviews to weed out employees who are a bad match for the firm, then it will fail to do so if the employees have low outside options. However, the problem does not persist. If the outside options increase then the incentives for the ill-matched type of the agent to try and blend in with the "good" type disappear and ill-matched types can easily be weeded out by the review.

I also show that changing the frequency of reviews doesn't change the main result - for low outside options more frequent reviews fail just as intermittent reviews fail. More frequent review does not always fare better than intermittent review. The quality of information has a significant bearing on the comparison of the two review regimes. This finding has implications for the design of performance management policies. In industries where outside options are low, either because they employ low skill labor or because the skills learned on the job are not transferable to a different job, this can be a serious concern. Performance reviews might not work, no matter how often they are conducted and the firm might do well in searching for a more effective mechanism to assess employee performance.

Chapter 3

Managerial effort and selection of risky projects under limited liability and risk neutrality

3.1 Introduction and motivation

Firms often hire executives to make decisions on their behalf. In different environments this leads to many interesting outcomes, several of which have been studied in the contract theory literature. In this chapter, I study one such environment. A firm is faced with the task of choosing between a risky and a safe project. There exists information that can help the firm rank the projects, but it cannot learn this information without hiring an executive. With enhanced information it might be possible to rank projects on a case by case basis and this gives rise to gains from information acquisition. Further, it is prohibitively costly[1] for the executive to communicate this information to the firm. It, thus, needs to delegate the task of learning about the project and subsequently choosing the project to the executive. Situations like this arise in multiple contexts. For example, one of the tasks of hedge fund managers is

[1]One of the alternative scenarios studied in the literature is when the executive *can* communicate the information to the firm, but might need the right incentives to report it correctly. See DeMarzo et al. (2013) for a treatment of this problem.

to learn about the investments and pick the right set on behalf of investors.

The key feature of this environment is that the firm wishes to delegate two interrelated tasks - learning new information and using the information correctly. The first task is costly to the executive and the second task is not[2]. Earlier papers have shown that this problem suffers from twin moral hazard - in the costly as well as the costless task. The latter is surprising and creates an interesting set of issues, which I attempt to address with this work.

The model used in this paper is very similar to the one originally analyzed in Lambert (1986). The paper showed that when the agent is risk averse and the principal is risk neutral, the agent may take excessive or insufficient risk. Put another way, despite learning the information, the agent picks different projects than what the principal would want him to under the optimal contract. Hellwig (2009) argues that it might not be possible to implement the first-best when there is a simultaneous risk and effort choice in a moral hazard setting despite the fact that both contracting parties are risk neutral. What then is the role of risk aversion in this context? I start by reconciling the two findings. Can the first-best be implemented if the agent is risk neutral? I find that a linear contract is able to implement the first-best outcome. Thus, Lambert's result is driven by risk aversion rather than the effect described in Hellwig (2009). This is exactly in line with the findings of Grossman and Hart (1983).

The focus of the chapter, however, is the impact of having a limited liability constraint for the agent. In most situations in which the agent assumes decision making responsibilities on behalf of the principal, the agent has limited liability. I study the problem in the presence of limited liability (and risk neutrality) and find that, once again, there may not exist a contract that achieves first best. In fact, the agent often chooses to take excessive/insufficient risk. Thus, limited liability not

[2]This is a simplifying assumption that allows me to detract from the additional incentives required to motivate costly effort to execute projects.

only raises the cost of the contract for the principal, it alters risk taking incentives. Several papers before this have discussed this effect. In particular, they have shown that limited liability constraints make risk taking more attractive. I discuss them in the next section. However, in the planning and implementation problem of Lambert this leads to a unique outcome. In particular, depending on the parameter values, the agent either takes excessive or insufficient risk.

The rest of the chapter is organized as follows. I start by discussing the existing literature. Then, I describe the model, while pointing out the similarities and differences between my model and that of Lambert. I define the agent and the principal's problems. Subsequently, I drop the limited liability constraint and show that there exists an optimal contract that implements the first-best outcome. The contract is linear and induces the agent to choose the same projects as the principal would have. Next, I use the first order approach to study some features of the optimal contract when there is a limited liability constraint. I show that if the first-best outcome is such that the risky project is more likely to be chosen by the principal under full information, then the agent takes insufficient risk. The contract in this case is concave. On the other hand, if the first-best outcome is such that the risky project is less likely to be chosen under full information, then the agent takes excessive risk. The contract is convex. The final section concludes and discusses some implications of these results as well as the caveats.

3.2 Relationship to literature

The interest in delegating the task of acquiring information goes back to Demski and Sappington (1987), who study the issues involved in hiring an expert to make decisions on the principal's behalf. The paper studies information acquisition and information use (implementation) in the presence of costly information. The focus of the paper is the relationship between the moral hazard that arises in the information acquisition phase and motivating action in the implementation phase. Since information acquisition is costly, moral hazard along that dimension is not unexpected. However, as Demski and Sappington point out, the implementation phase is costless to the agent. The fact that there is moral hazard in the second agent choice (implementation), which is costless to the agent, is the central finding of this paper. The underlying reason behind the difficulty in creating incentives for obtaining information *and* implementation is the difference in the risk attitudes of the principal and the agent. Hirshleifer and Suh (1992) extend the problem by studying the importance of the curvature of the agent's contract as a function of profit in providing incentives to the agent to plan and implement. Once again, the paper studies the impact of multiple model features - risk aversion of agent, the relationship of payoff and effort during implementation. They find that the level of risk imposed on the agent by the contract depends on the availability of risky growth opportunities and on the effectiveness of monitoring institutions. This chapter demonstrates that non linear contracts can arise in an environment with risk neutrality. The trade-off between effort and risk has also been studied by Hellwig (2009) who argues that it is not possible to implement the first-best outcome when such a trade-off exists in conjunction with moral hazard. The agent solicits funds from the principal. He then chooses how risky a project should be and how much effort to give to the project. This paper shows that when the choice of riskiness and effort are simultaneously unobservable then the first best can't be implemented. The paper also talks about the importance of technology when it comes to examining double moral hazard. The elasticity of substitution between effort and investment turns out to be very important in determining the nature and intensity of agency problems.

This chapter is also closely related to the literature on the impact of limited liability. The central finding of this literature is that limited liability constraints don't just make contracting expensive, they fundamentally change risk taking incentives. Sappington (1983) studies the impact of limited liability on principal agent relationships. The author finds that it becomes optimal for the principal to induce efficient actions only in the most productive and least productive states of nature. This is because he disproportionately bears the downside risk. In a related paper, Innes (1990) studies the one dimensional moral hazard problem where an entrepreneur (agent) seeks to design a contract that attracts investors (principals) for a project that require him to make costly effort under limited liability. Just as in this chapter, both the principal and agent are assumed to be risk neutral to detract from risk sharing issues. In the absence of limited liability, a linear contract would be sufficient for motivating the first-best outcome. However, limited liability makes risk taking more attractive. In response to this excessive risk taking incentive, the agent designs a contract that limits his own moral hazard by bearing as much risk as possible by using debt contracts or live or die contracts. This makes the investment attractive to investors. Hébert (2014) extends and generalizes these results. DeMarzo et al. (2013) studies the impact of limited liability in the presence skimming - the principal does not observe the payoff, the agent reports it and can under report if he wants. In the static model, the agent can choose between a safe project or a risky project. The safe project gives a return of 1 or 0. The risky project can give 1 with a higher probability, 0 with a lower probability and with some probability causes a disaster. In addition, the agent can skim from the returns. The principal wants to construct a contract that provides incentives to choose the safe project and at the same time report the return honestly. He needs to give high wages for reporting a high payoff or the agent will skim. That creates incentives to gamble and the principal can't punish the agent for gambling because of limited liability. So he has to give the agent some transfer for zero outcomes to distinguish them from disasters. Thus, the principal can induce the safe outcome and even get truthful reporting, but it is costly. If instead, the principal can condition the contract on observing a disaster, he can reward the agent for all non disaster outcomes and that makes it cheaper for the principal to implement his chosen actions. So basically, if there is a disaster and the firm gets a zero it is optimal to give the agent a bonus. This is cheaper than giving the agent a positive transfer to zero output always.

As mentioned in the introduction, the work in this chapter is most closely related to Lambert (1986). I start by showing that Lambert's results regarding excessive/insufficient risk-taking in a delegating expertise problem (information acquisition and use) depend on the risk attitude of the agent. In particular, the fact that he is risk averse. Then I drop the risk aversion to detract from risk sharing concerns and add a limited liability constraint. In line with the findings of previous literature, I also find that limited liability does more than just raise the cost of the contract. Limited liability changes changes the risk taking incentives in the model. Depending on the parameter values, it might lead to excessive or insufficient risk taking. To the best of my knowledge, this effect has not been documented in earlier literature. I discuss the intuition behind this result and discuss the caveats of the analysis.

3.3 Model

There are two projects. One project is safe and it yields a fixed output, x_0. The other project is risky and can yield a high (x_H) or a low (x_L) output, where $x_L < x_0 < x_H$. In the absence of any effort, the common prior belief is that the risky project yields x_H with probability 0.5. However, there exists an option to exert effort to get information about the risky project. The cost of effort[3] is V. Upon exerting effort, the posterior probability of the risky project becomes known - the risky project yields x_H with probability p, where $p \sim U[0, 1]$. Notice, that the safe project is not always preferable

[3] This can alternatively be thought of as the cost of the information itself.

to the risky project, or vice versa. The principal cannot observe p and must delegate the task to an agent who has to decide whether or not to exert effort, and subsequently which project to choose. Assume that the principal and the agent are risk neutral. In addition, the agent has limited liability[4].

Suppose the principal offers the agent a wage contract, $w = (w_0, w_H, w_L)$. If the agent accepts the contract, exerts no effort and the output is x_i, then he gets w_i. If he does exert effort he gets $w_i - V$. If he rejects the contract he gets θ.

3.3.1 The first-best outcome

If the principal could observe whether or not the agent worked and also the p that the agent observed, he could make the contract contingent upon working and making the right decision. The principal wants the agent to pick the risky project as long as $px_H + (1-p)x_L \geq x_0$. This gives a cut off value for p, say p_f (first best), such that the principal wants the agent to choose the risky project whenever $p \geq p_f$ and the safe one otherwise. The cutoff value is given by:

$$p_f = \frac{x_0 - x_L}{x_H - x_L}$$

With this cutoff, the total probability of observing x_0, x_H and x_L is given by p_f^0, p_f^H and p_f^L respectively, where

[4] In Lambert (1986), the agent is risk averse and there is no limited liability. All other features of the environment are identical.

$$
\begin{aligned}
p_f^0 &= \int_0^{p_f} f(p)dp = p_f \\
p_f^H &= \int_{p_f}^1 pf(p)dp = 0.5(1-p_f^2) \\
p_f^L &= \int_{p_f}^1 (1-p)f(p)dp = 0.5(1-p_f)^2
\end{aligned}
\tag{3.1}
$$

Denote by $E_f X = \left[p_f^0 x_0 + p_f^H w_H + p_f^L w_L\right]$. Notice that the firm can always do better by finding out p than it can by picking the safe or the risky project without this information because $\pi_I = E_f X - \max\{x_0, 0.5(x_H + x_L)\} > 0$. π_I can be thought of as the value of information. Put another way, if the information was free, the firm would always want to make the informed decision rather than the uninformed decision. Let $E_{UI}X = \max\{x_0, 0.5(x_H + x_L)\}$. $E_{UI}X$ is the highest expected payoff that can be obtained by making an uninformed choice. The principal will want the agent to exert effort as long as exerting effort and choosing the cutoff p_f yields a higher payoff than not exerting effort and choosing the safe or the risky project. That is

$$
E_f X - \max\{x_0, 0.5(x_H + x_L)\} \geq V \tag{3.2}
$$

I assume throughout that (3.2) holds.

Under the assumption of full information, the principal pays the agent a risk free wage $V + \theta$, conditional on the agent working and choosing the cutoff p_f, and 0 otherwise.

Suppose the agent could costlessly (and truthfully) communicate the information he learns to the principal. Then the principal only delegates the task of learning the information to the agent and always picks cutoff p_f. He could then pay the agent a flat wage of $V + \theta$ and get the first-best outcome. Choosing the project is costless irrespective of who does it. As the analysis that follows shows, delegating this costless task to the agent makes it impossible to achieve the first-best outcome for some parameter values. This phenomenon is what Demski and Sappington (1987) call spillover moral hazard.

3.3.2 Moral hazard

The interesting case, however, is when the principal cannot observe agent effort and conditional on the agent making the effort, the posterior probability of x_H. The moral hazard problem arises from the fact that even though the principal can observe the output, he can't, for instance, distinguish between the following three situations:

1. The agent does not work and chooses the safe project.
2. The agent works and chooses the safe project knowing that it yields a higher output than the expected output of the risky project.
3. The agent works and chooses the safe project even though it yields a lower output than the expected output of the risky project.

Suppose the principal offers the agent $w \in \mathcal{R}^3_+$, the agent chooses to accept the contract, and subsequently to work. As long as $w_H \geq w_L$ the expected value of picking the risky project, $pw_H + (1-p)w_L$, is increasing in p. The agent maximizes his payoff by picking a cutoff value of p, say $\hat{p}(w)$, given by

$$\hat{p}(w) = \frac{w_0 - w_L}{w_H - w_L} \tag{3.3}$$

For all $p \leq \hat{p}(w)$ the agent will choose the safe project, and the risky project otherwise. I limit my search to contracts[5] in which $w_H \geq w_L$.

This cutoff gives rise to probabilities $p^0(w), p^H(w), p^L(w)$ of observing outcomes x_0, x_H, x_L respectively, where

$$\begin{aligned} p_0(w) &= \int_0^{\hat{p}(w)} f(p)dp = \hat{p}(w) \\ p_H(w) &= \int_{\hat{p}(w)}^1 pf(p)dp = .5(1-\hat{p}(w)^2) \\ p_L(w) &= \int_{\hat{p}(w)}^1 (1-p)f(p)dp = .5(1-\hat{p}(w))^2 \end{aligned} \tag{3.4}$$

Using the above probabilities the principal chooses the contract to solve the following problem:

$$\max_{(w_0,w_H,w_L)\in\mathbb{R}^3_+} p^0(w)(x_0 - w_0) + p^H(w)(x_H - w_H) + p^L(w)(x_L - w_L) \tag{3.5}$$

subject to

$$p^0(w)w_0 + p^H(w)w_H + p^L(w)w_L - V \geq \theta \tag{3.6}$$

$$p^0(w)w_0 + p^H(w)w_H + p^L(w)w_L - V \geq 0.5w_H + 0.5w_L \tag{3.7}$$

$$p^0(w)w_0 + p^H(w)w_H + p^L(w)w_L - V \geq w_0 \tag{3.8}$$

The constraints ensure that the agent accepts the contract (3.6), chooses to work rather than shirk and always pick the risky (3.7) or the safe project (3.8). I refer to the above problem as *formulation 0*. It turns out that formulation 0 can be restated to make it more tractable. Later in the chapter I do so and call that *formulation 1*.

[5]Contracts in which $w_L < w_H$ will induce the agent to pick a $\hat{p}(w)$ such that, for all $p \leq \hat{p}(w)$, he will pick the risky project.

Let w_0^*, w_H^*, w_L^* be an optimal contract and p_s (second best) be the cutoff value induced by this contract. I start by noting three characteristics of optimal contracts.

First, $p_s \neq \{0,1\}$. Suppose $p_s = 0$, then $p^0(w^*) = 0$ and $p^H(w^*) = p^L(w^*) = 0.5$. Constraint (3.7) then becomes $-V \geq 0$, which violates the assumption that $V > 0$. A similar argument can be used to show that $p_s \neq 1$. Setting $p_s = 1$ would violate constraint (3.8). In later proofs, this fact is used extensively.

Second, in any optimal contract $w_0^* \in [w_L^*, w_H^*]$. Suppose $w_0^* > w_H^*, w_L^*$. Then, $w_0^* > p^0(w^*)w_0^* + p^H(w^*)w_H^* + p^L(w^*)w_L^*$ and $w_0^* > p^0(w^*)w_0^* + p^H(w^*)w_H^* + p^L(w^*)w_L^* - V$. This violates (3.8). Suppose instead $w_0^* < w_H^*, w_L^*$ then $0.5w_H^* + 0.5w_L^* > p^0(w^*)w_0^* + p^H(w^*)w_H^* + p^L(w^*)w_L^*$ and $0.5w_H^* + 0.5w_L^* > p^0(w^*)w_0^* + p^H(w^*)w_H^* + p^L(w^*)w_L^* - V$ violating (3.7). Thus, $w_H^* \geq w_0^* \geq w_L^*$. Hence, the principal's problem only needs one limited liability constraint, $w_L \geq 0$.

Finally, not all w_i^*'s are equal. That is, the principal doesn't pay the agent a flat wage in any optimal contract. If $w_H^* = w_0^* = w_L^*$ then constraints (3.8) and (3.7) are violated. This, along with the fact that $w_H^* \geq w_0^* \geq w_L^*$, implies that $w_H > w_L$, which will be used later.

The rest of the analysis is organized as follows. First, I characterize the optimal contract when the agent does not have limited liability. I do so by solving for the optimal linear contract and then arguing that the optimal linear contract is optimal among all contracts. Subsequently, I return to the case where the agent has limited liability. Under some conditions on the parameters the above result still holds, but it is not true for all parameter values. When the optimal linear contract fails to be optimal in general, I study and discuss some characteristics of the optimal contract.

3.3.3 Optimal contract in the absence of limited liability

Consider the principal's problem without the limited liability constraint for the agent. Suppose, the principal restricts his attention to contracts of the form:

$$w_i = \alpha x_i + \beta \tag{3.9}$$

where $\alpha \geq 0$ and $\beta \in \mathcal{R}$. Notice from (3.3) that with a linear contract the agent always uses the first-best cut off p_f. That is, $\hat{p}(w) = p_f$ if w is given by (3.9) and $\alpha \neq 0$. Then, the probabilities $p^0(w)$, $p^H(w)$ and $p^L(w)$ are given by (3.1).

The principal's problem is given below and represented by figure Figure 3.1. The dotted line represents the set of (α^*, β^*) that are optimal.

$$\max_{\alpha \geq 0, \beta \in \mathcal{R}} (1-\alpha) E_f X - \beta \tag{3.10}$$

$$\text{subject to}$$

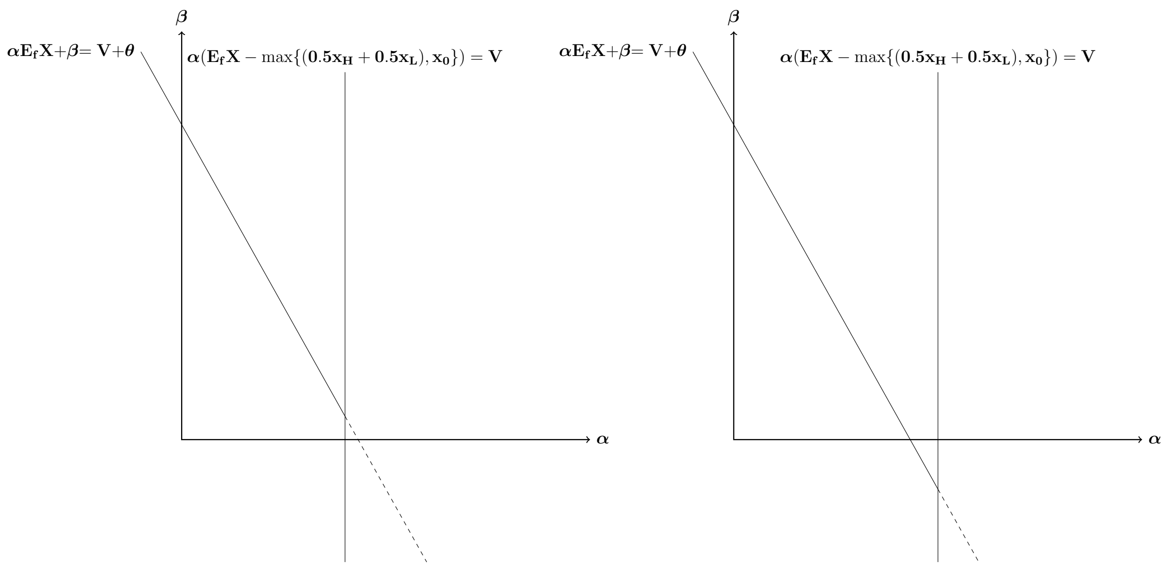
$$\alpha E_f X + \beta \geq V + \theta \tag{3.11}$$

$$\alpha(E_f X - \max\{(0.5x_H + 0.5x_L), x_0\} \geq V \tag{3.12}$$

Maximizing the objective function (3.10) is equivalent to minimizing $\alpha E_f X + \beta$. At the optimum linear contract the participation constraint (3.11) holds with equality and inequality (3.12) holds. The optimal contract (α^*, β^*) satisfies the following conditions:

$$\alpha^* E_f X + \beta^* = V + \theta \tag{3.13}$$

$$\alpha^*(E_f X - \max\{(0.5x_H + 0.5x_L), x_0\} \geq V \tag{3.14}$$



(a) Case I (b) Case II

Figure 3.1: Optimal Linear Contract without limited liability

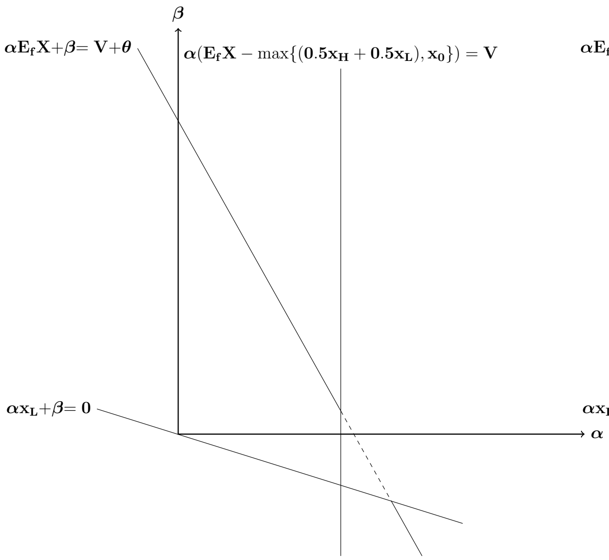
The optimal contract implements the first-best cutoff and the expected payoff to the agent is $V + \theta$. As a result, there is no contract that can do strictly better. This gives the following proposition.

Proposition 3.1. *In the absence of limited liability, there exists an optimal contract which is linear.*

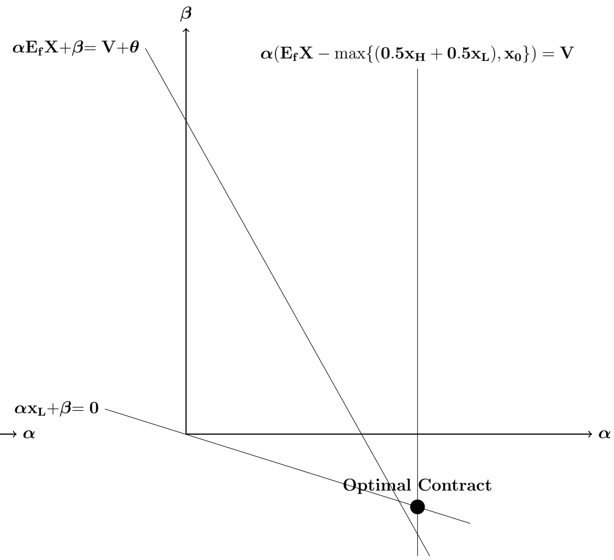
Further, the optimal contract has the following characteristics.

- If $\frac{V}{\theta} \geq \frac{\pi_I}{E_{UI}X}$ then it is possible to find (α^*, β^*) such that $\beta^* \geq 0$
- If $\frac{V}{\theta} < \frac{\pi_I}{E_{UI}X}$ then $\beta^* < 0$ for all optimal contracts that are linear.

Thus, Lambert's result regarding excessive/insufficient risk-taking is due to risk aversion. The optimal contract induces the first-best outcome if the agent is risk



(a) Case I



(b) Case II

Figure 3.2: Optimal Linear Contract when $x_L > 0$

neutral. However, that is not to say that his result is a peculiar feature of having a risk averse agent. It turns out that adding limited liability generates a similar result.

Under limited liability the agent may capture some of the surplus if the principal restricts his attention to linear contracts. To see this, I solve for the optimal linear contract in the problem with limited liability. This contract would have to solve the problem given by (3.10)-(3.12) along with the limited liability constraint $\alpha x_L + \beta \geq 0$. Let $(\alpha^*_{LL}, \beta^*_{LL})$ be an optimal contract under limited liability. Figure Figure 3.2 represents the case in which $x_L > 0$, however the analysis is the same for the cases in which $x_L \leq 0$. When $x_L < 0$, the line $\alpha x_L + \beta = 0$ is upward sloping and $\beta^*_{LL} > 0$ for all optimal linear contracts. The dotted line represents the set of (α^*, β^*) that are optimal in case I and in case II the optimal linear contract is a singleton. As long as $\frac{V}{\theta} \leq \frac{\pi_I}{(E_{UI}X - x_L)}$, the participation constraint holds with equality and there exists an optimal contract that is linear. However if

$$\boxed{\frac{V}{\theta} > \frac{\pi_I}{(E_{UI}X - x_L)}} \tag{3.15}$$

then the participation constraint does not hold with equality in the optimal linear contract. The optimal linear contract need not be optimal in general.

Thus, under some conditions the principal has to pay more than $V + \theta$ when limited liability is imposed in the problem if he continues to use linear contracts. This observation is in line with DeMarzo, 2013 DeMarzo et al. (2013) where the authors show that the presence of a limited liability constraint makes the optimal contract expensive for the principal. The size of this premium is given by the difference between what the principal pays out in the optimal contract and $V + \theta$ and is denoted by P_{LL}

$$P_{LL} = V\left(\frac{E_{UI}X - x_L}{\pi_I}\right) - \theta$$

The above expression is strictly greater than zero whenever the participation constraint does not hold with equality in the optimal linear contract.

One of the advantages of studying linear contracts in this problem is that it becomes possible to sidestep the fact that the objective function is not easy to work with in formulation 0. This is one of the major differences between the problem at hand and standard project choice problems with moral hazard. The presence of a cutoff point introduces nonconvexities in the principal's optimization problem. In the next subsection I restate the problem to a form that is easier to work with. While the new formulation continues to suffer from the same nonconvexities, it is possible to get several insights from the first order conditions. I discuss this is greater detail in the appendix.

3.3.4 Optimal contract with limited liability

The next issue that this chapter seeks to address is the nature of the optimal contract in the presence of limited liability. As shown in the previous section, in the presence of limited liability the optimal linear contract may not be optimal in general. In order to do so, I study the first order conditions of the principal's optimization problem. However, the objective function and the constraints in formulation 0 yield first order conditions that are difficult to analyze because the total probability of observing each outcome is a function (sometimes quadratic function) of $\frac{w_0 - w_L}{w_H - w_L}$. In order to make the first order conditions easier to analyze, I reformulate the problem by making $\hat{p}$ one of the choice variables along with $(w_), w_H, w_L)$. This approach was also used in Lambert (1986).

As before, suppose the principal offers the agent a wage contract $w = (w_0, w_H, w_L) \in \mathcal{R}^3_+$ and the agent chooses a cutoff value $\hat{p}$ such that for all $p \leq \hat{p}$ he picks the safe project. Then, the total probability of observing outcomes x_0, x_H and x_L as a function of $\hat{p}$ is calculated using equations analogous to (3.4) and is given by $p_0(\hat{p})$, $p_H(\hat{p})$ and $p_L(\hat{p})$ respectively.

The agent chooses $\hat{p}$ to maximize

$$p_0(\hat{p})w_0 + p_H(\hat{p})w_H + p_L(\hat{p})w_L$$

This yields the first order condition

$$w_0 - (\hat{p}w_H + (1 - \hat{p})w_L) = 0$$

The above equation becomes one of the constraints in the principal's optimization problem when he chooses the agent's contract. The principal's problem can then be rewritten. I call this statement of the problem *formulation 1.*

$$\max_{(w_0,w_H,w_L)\in\mathbb{R}^3,\hat{p}\in[0,1]} p^0(\hat{p})(x_0 - w_0) + p^H(\hat{p})(x_H - w_H) + p^L(\hat{p})(x_L - w_L) \tag{3.16}$$

subject to

$$p^0(\hat{p})w_0 + p^H(\hat{p})w_H + p^L(\hat{p})w_L - V \geq \theta \tag{3.17}$$

$$p^0(\hat{p})w_0 + p^H(\hat{p})w_H + p^L(\hat{p})w_L - V \geq w_0 \tag{3.18}$$

$$p^0(\hat{p})w_0 + p^H(\hat{p})w_H + p^L(\hat{p})w_L - V \geq 0.5w_H + 0.5w_L \tag{3.19}$$

$$w_0 - [\hat{p}w_H + (1-\hat{p})w_L] = 0 \tag{3.20}$$

$$w_L \geq 0 \tag{3.21}$$

Let $\lambda, \mu_1, \mu_2, \eta, \xi$ be the lagrange multipliers for constraints (3.17)-(3.21). In what follows, I use p_0, p_H, p_L in lieu of $p_0(\hat{p}), p_H(\hat{p}), p_L(\hat{p})$. Let $\lambda^*, \mu_1^*, \mu_2^*, \eta^*, \xi^*$ be the values of the multipliers satisfying the first order conditions if the wage is w^*. Further, let p_0^*, p_H^*, p_L^* be the values of $p_0(\hat{p}), p_H(\hat{p}), p_L(\hat{p})$ when $\hat{p} = p_s$. Note that, while $\lambda^* \geq 0, \mu_1^* \geq 0, \mu_2^* \geq 0$ and $\xi^* \geq 0$, η^* can be positive, negative or zero.

In the appendix I prove several technical results about the multipliers. However, the focus is on establishing the characteristics of η^* and ξ^*. In particular, I show that the sign of η^* is closely related to excessive/insufficient risk-taking in the model. For instance, lemma B.3 shows that if $\eta^* > 0$, then $p_s < p_f$. This relationship is then used to show that if $p_f > 0.5$ then $p_s \in [0.5, p_f]$ and the agent takes excessive risk under the optimal contract. Further, the optimal value of ξ^* sheds light on whether or not the limited liability constraint holds with equality in the optimal contract. I show that η^* and ξ^* are either both zero or both non zero. Thus, contracts in which the agent takes excessive/insufficient risk are also contracts in which the limited liability constraint holds with equality.

The first order conditions of the principal's problem are given below[6].

$$1 = \lambda^* + \mu_1^* \frac{p_0^* - 1}{p_0^*} + \mu_2^* + \eta^* \frac{1}{p_0^*} \tag{3.22}$$

$$1 = \lambda^* + \mu_1^* + \mu_2^* \frac{p_H^* - 0.5}{p_H^*} - \eta^* \frac{p_s}{p_H^*} \tag{3.23}$$

$$1 = \lambda^* + \mu_1^* + \mu_2^* \frac{p_L^* - 0.5}{p_L^*} - \eta^* \frac{(1 - p_s)}{p_L^*} + \xi^* \frac{1}{p_L^*} \tag{3.24}$$

$$\Sigma_{j \in \{0,H,L\}} (x_j - w_j^*) \frac{\partial p_j(\hat{p})}{\partial \hat{p}} |_{\hat{p} = p_s} - \eta (w_H^* - w_L^*) = 0 \tag{3.25}$$

Proposition 3.2 is one of the central results of this paper. It shows that if $p_f < 0.5$ the agent takes insufficient risk by choosing a cutoff that is higher than p_f. All proofs and lemmas are relegated to the appendix.

Proposition 3.2. *a) If* $p_f < 0.5$ *then* $p_s \in [p_f, 0.5]$.

b) If $p_f > 0.5$ *then* $p_s \in [0.5, p_f]$.

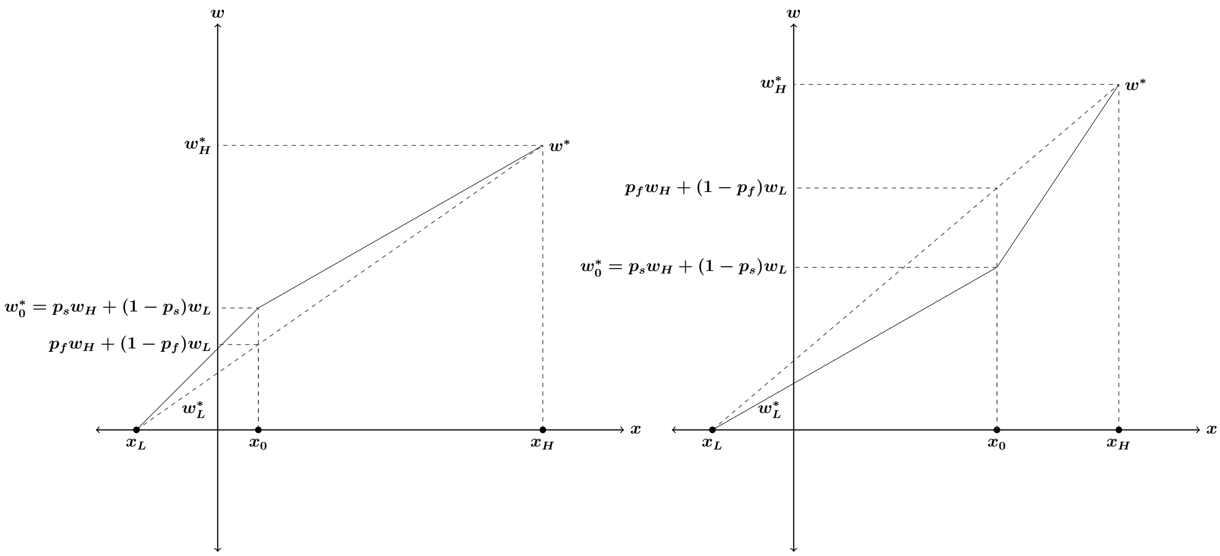
c) If $p_f = 0.5$ *then* $p_s = p_f$.

When $p_f < 0.5$, then under the uninformed prior the risky project yields a higher expected payoff than the safe project. Thus, the contract needs to motivate the agent to work rather than shirk and choose the risky project. To motivate the agent to work the contract must make the safe project *sufficiently* attractive. Suppose we start by dropping the limited liability constraint and picking an optimal linear contract. This would make the safe project sufficiently attractive, the agent picks the cutoff p_f and the expected cost of the contract to the principal would be $V + \theta$. Now let's add back the limited liability constraint such that the optimal linear contract no longer satisfies it and w_L has to be raised to 0, thus raising the payoff from the risky project. The

[6]The objective functions ((3.5), (3.16)) in both the formulations of the given problem are neither concave nor quasiconcave (see appendix).

agent would then start taking excessive risk, picking $\hat{p} < p_f$. In addition, this contract costs the principal more than $V+\theta$, say $V+\theta+\epsilon$. The principal can do strictly better by raising w_0 and decreasing w_H while keeping the costs $V+\theta+\epsilon$ and increasing the cut off back to $\hat{p} = p_f$. Can the principal do even better? Perhaps, he can. He could continue to use a combination of raising w_0 and decreasing w_H such that $\hat{p} > p_f$, while decreasing the expected cost of the contract to some value between $V+\theta+\epsilon$ and $V+\theta$. At this point, the principal faces the trade-off between decreasing the cost of the motivating effort and decreasing the expected payoff from the project by raising $\hat{p}$ above p_f. Proposition 3.2 shows that the two effects (motivating work and reducing cost of contract while ensuring that the participation constraint is satisfied) interact in a way to the optimal cutoff is higher than p_f. The principal would rather have the agent work and take insufficient risk than have him not work at all or pay him a very large amount to work and pick the first-best cut off. If $p_s \geq p_f$, then $w_0 = p_s w_H + (1-p_s)w_L \geq p_f w_H + (1-p_f)w_L$ and the contract is concave (see figure Figure 3.3).

Consider the alternative case where $p_f > 0.5$. The expected payoff from the risky project under the prior distribution is then lower than payoff of the safe project. Now the principal needs to make the risky project *sufficiently* attractive. Once again, I start from an optimal linear contract without limited liability and imposing a limited liability constraint, the expected cost of the contract to the principal is higher than $V+\theta$ and the agent uses a cutoff that is lower than p_f, thus taking excessive risk. Once again, keeping the costs constant (and higher than $V+\theta$), the principal could do strictly better by raising w_0 and/or dropping w_H such that the cutoff used by the agent becomes p_f once again. However, the principal can do potentially even better by lowering costs. The only way that principal can lower costs is by lowering w_0 and/0r w_H. Proposition 3.2 shows that the two effects (motivating work and reducing cost of contract while ensuring that the participation constraint is satisfied)



(a) $p_f < 0.5$ (b) $p_f > 0.5$

Figure 3.3: Concavity and convexity of optimal contract

interact in a way to the optimal cutoff is lower than p_f. The principal would rather have the agent work and take excessive risk than have him not work at all or pay him a very large amount to work and pick the first-best cut off. If $p_s \leq p_f$, then $w_0 = p_s w_H + (1-p_s) w_L \leq p_f w_H + (1-p_f) w_L$ and the contract is convex (see figure Figure 3.3).

In what follows, I show that contracts in which the agent takes excessive/insufficient risk are also contracts in which the limited liability constraint holds with equality.

Proposition 3.3. *Suppose $p_s \neq 0.5$. Then, $p_s \neq p_f$ if and only if the limited liability constraint is binding*[7].

Proposition 3.3 is particularly useful for the case where the optimal linear contract is not optimal among all contracts. If $p_s \neq 0.5$, whenever $p_s \neq p_f$, the multiplier on

[7] Actually, the more precise statement would be that the multiplier for the limited liability constraint is strictly positive. This of course implies that the constraint holds with equality. However, of course, these are not identical statements.

the limited liability constraint is strictly positive and $x_L = 0$. Conversely, whenever the multiplier on the limited liability constraint is strictly positive, $p_s \neq p_f$. Thus, the presence of a limited liability constraint has a significant impact on the level of investment in this model.

I bring the chapter to a close by returning to the case when optimal linear contract is not optimal among all contracts. Recall that if $\frac{V}{\theta} > \frac{\pi_I}{(E_{UI}X - x_L)}$, then the optimal linear contract may not be optimal across all contracts. Proposition 3.3 further implies that the optimal linear contract is *not* optimal across all contracts.

Corollary 3.1. *If* $\frac{V}{\theta} > \frac{\pi_I}{(E_{UI}X - x_L)}$ *and* $p_f \neq 0.5$, *then the optimal contract is not linear.*

Corollary 3.1 completes the characterization of contracts in this model. To summarize:

1. If there is a limited liability constraint and $p_f = 0.5$, then $p_s = 0.5$ and the optimal contract is linear. Note, that it need not be one in which the principal pays $V + \theta$.

2. If $p_f \neq 0.5$

   - If $\frac{V}{\theta} \leq \frac{\pi_I}{E_{UI}X - x_L}$, once again, there exists an optimal contract that is linear. The expected payoff to the agent is $V + \theta$

   - If $\frac{V}{\theta} > \frac{\pi_I}{E_{UI}X - x_L}$ and $p_f \neq 0.5$ then the optimal contract is not linear. There must be a non linear contract that is optimal. Then

     a) If $p_f < 0.5$ then $p_s \in (p_f, 0.5]$.

     b) If $p_f > 0.5$ then $p_s \in [0.5, p_f)$

     c) $x_L = 0$ i.e. the limited liability constraint on x_L holds with equality.

3.4 Conclusion and discussion

This chapter studies the characteristics of the optimal contract when a risk neutral principal delegates the task of choosing between a safe and a risky project to to a risk neutral agent. The agent needs to exert costly effort to find information about the riskiness of the project and subsequently choose the project. I find that the principal uses a linear, concave or convex wage contracts under different conditions. Concave contracts lead to insufficient risk taking and convex contracts lead to excessive risk taking.

The results in this paper depend crucially on the assumption that the posterior probability is distributed uniformly. This assumption guarantees that the first order condition of the agent's problem when he chooses the cutoff value $\hat{p}$ is sufficient. This feature makes it possible to rewrite the principal's problem (formulation 0) such that the first order conditions are tractable (formulation 1). Further, the assumption implies that the expected payoff of the project when making an informed choice is strictly concave. This is essential for establishing the relationship between p_f and p_s (see proof of proposition 3.2). Extending this analysis to other posterior distributions would be an interesting line of research.

As mentioned earlier the objective function of the principal's problem is neither concave nor quasiconcave. The first order conditions (3.22)-(3.25) are not sufficient. However, it is straightforward to argue that a solution to the problem exists. The objective function (3.5) is continuous. The constraint set is not compact, however it can be made compact without altering the set of optimal solutions. Suppose, I add the constraints that $w_0, w_H, w_L \leq x_H + x_0 + \epsilon$, $\epsilon > 0$ to the principal's problem. The maximum surplus that the principal can get from any project in any state of the world is x_H. Suppose the principal accepts a negative surplus in one state in lieu of a positive surplus in other states such that the value of his objective function is at least zero[8]. The maximum negative surplus he would be willing to accept is $x_H + x_0$, because that is the maximum that he can recoup as a positive surplus in the other states. Recall that in all optimum contracts $p_s \neq \{0, 1\}$. Thus the constraints $w_0, w_H, w_L \leq x_H + x_0 + \epsilon$, $\epsilon > 0$ will never hold with equality and hence imposing them will not alter the set of optimum solutions. These constraints do, however, make the constraint set compact. Now, using the extreme value theorem the principal's optimization problem has a solution.

Numerous extensions of the analysis in this essay could be made by future research. It would be fruitful to examine alternative distributions from which p could be drawn that would be able to generate similar results. The characteristics of this set of distributions could potentially shed light of the impact of information acquisition devices on risk taking. Another possible extension is to examine the potential impact of communication devices in this model. As argued earlier, information use itself is costless and if it could be freely communicated to the principal there would be no moral hazard. Introducing a communication device, even an imperfect or costly one, could potentially decrease the wedge between the first and second best outcomes.

[8]If the value of the objective function is negative, then that wage contract can not be the optimum because the principal can do better by not hiring the agent.

Chapter 4

Managerial effort and selection of risky projects over time

4.1 Introduction and motivation

Most employee-employer relationships involve repeated interactions over time. In some situations this can make it easier to implement the full information outcome. However, there are many ways in which this repeated interaction can be organized. In this chapter, I study the problem of implementing the first-best outcome when the task described in chapter 3 is delegated to the agent for two periods rather than one. I compare three contracting regimes - period-by-period contracting, two-period contracting with no recontracting at date-1 and two-period contracting with recontracting. I restrict my attention to the conditions under which the first best can be implemented. Allowing the agent to recontract means allowing him to access his outside option at date-1.

In this chapter, I show that contracting at date-0 for two periods while allowing for recontracting yields the widest range of parameter values under which the first best can be implemented. This implies that there exists a range of parameter values for which it is not possible to implement the first-best outcome under period-by-period contracting, however it is possible to do so under two-period contracting with recontracting. This suggests that contracting at date-0 for two periods while allowing for recontracting offers an advantage over contracting period-by-period. I also derive the conditions under which two-period contracting without recontracting is superior to period-by-period contracting.

In chapter 3 the principal delegates the task of acquiring information and using it to choose between a risky and a safe project to the agent. If

$$\frac{V(E_{UI}X - x_L)}{\pi_I} \leq \theta$$

the linear contract implements the first best and is optimal. If this condition does not hold, then the optimal contract is nonlinear and does not implement the first best. In this chapter, I extend the analysis to a dynamic setting and compare the threshold analogous to the one above for the three contracting regimes mentioned earlier.

4.2 Period-by-period contracting

Recall the principal's problem if he offers a linear contract to the agent for one period. Let θ_1 be the agent's outside option when he first starts working with the principal (date-0).

$$\max_{\alpha \geq 0, \beta \in \mathcal{R}} (1-\alpha)E_f X - \beta \tag{4.1}$$

subject to

$$\alpha E_f X + \beta \geq V + \theta_1 \tag{4.2}$$

$$\alpha(E_f X - \max\{(0.5x_H + 0.5x_L), x_0\} \geq V \tag{4.3}$$

$$\alpha x_L + \beta \geq 0 \tag{4.4}$$

Suppose the principal wants to hire the agent for a second period and the latter's outside option changes to θ_2. The contracting problem remains the same, with the exception of the participation constraint which is now $\alpha E_f X + \beta \geq V + \theta_2$. We know that in each of these one period problems there exists an optimal contract that is linear and implements the first best if

$$\frac{V(E_{UI}X - x_L)}{\pi_I} \leq \theta_i$$

Thus, if the principal and agent contracted period-by-period for two periods, there would exist an optimal contract that is linear and implements the first best if:

$$\frac{V(E_{UI}X - x_L)}{\pi_I} \leq \min\{\theta_1, \theta_2\} \tag{4.5}$$

4.3 Two-period contracting with no recontracting

Now suppose the agent chooses projects twice, however the principal and agent contract only once at date-0. Further, the agent is not allowed to exit the contract at date-1. The principal is able to observe the outcome for both periods at the end of date-1, but can't he tell if the agent worked and whether the choices were made using p_f as the cutoff rule. I call this complete observability. An alternative assumption would be that the principal observes a function of the output for the two periods and not necessarily the order in which the output was realized. A special case of this is that the principal observes the sum of the output in both periods. I refer to this situation as partial observability. Later in this section I argue that replacing the complete observability assumption with partial observability does not alter the result.

Suppose the principal offers the agent the following contract

$$w =\in \mathcal{R}_+^9 = \{w_{00}, w_{0H}, w_{0L}, w_{H0}, w_{HH}, w_{HL}, w_{L0}, w_{LH}, w_{LL}\},$$

where w_{ij} is the wage given to the agent when the output is $\{x_i, x_j\}$. The principal chooses w_{ij}'s such that the agent works in every period, and subsequently picks projects optimally. If the agent rejects the contract he gets θ_3 - the cumulative outside option for two periods.

In order to formulate the principal's problem I use backward induction. Consider the agent's decision at date-1. Notice that it doesn't matter for the date-1 decision, whether the date-0 outcome was a result of working or shirking. Suppose the agent observed outcome $i \in \{0, H, L\}$ at date-0. Then at date-1 the agent would choose the risky project as long as $p \geq \hat{p}_i(w)$ where

$$\hat{p}_i(w) = \frac{w_{i0} - w_{iL}}{w_{iH} - w_{iL}}$$

Using this cutoff, the probability of observing x_0, x_H, x_L at date-1 is given by $p_{i0}(w), p_{iH}(w), p_{iL}(w)$.

The principal constructs his contract in a way that the agent would rather work at date-1 than shirk and choose either the safe or the risky project. That is to say, for every $i \in \{0, H, L\}$

$$p_{i0}w_{i0} + p_{iH}w_{iH} + p_{iL}w_{iL} - V \geq \max\{w_{i0}, 0.5w_{iH} + 0.5w_{iL}\} \tag{4.6}$$

Taking one step back, let's consider the agent's date-0 decision. I define $\hat{U}_i(w)$ and $\hat{X}_i(w)$ for all $i \in \{0, H, L\}$. Let $\hat{U}_i(w)$ and $\hat{X}_i(w)$ be the date-1 continuation value of the payoff to the agent and principal respectively if the agent chooses according to (4.6) at date-1. That is:

$$\hat{U}_i(w) = p_{i0}w_{i0} + p_{iH}w_{iH} + p_{iL}w_{iL} - V$$
$$\hat{X}_i(w) = p_{i0}(x_i + x_0 - w_{i0}) + p_{iH}(x_i + x_H - w_{iH}) + p_{iL}(x_i + x_L - w_{iL})$$

From the point of view of the agent, the date-0 decision is exactly the same as the date-1 decision following any i, with the exception that instead of receiving $\{w_{i0}, w_{iH}, w_{iL}\}$ he will receive $\{\hat{U}_0(w), \hat{U}_H(w), \hat{U}_L(w)\}$. Let $\hat{p}(w)$ be the cutoff that the agent chooses at date-0, where

$$\hat{p}(w) = \frac{\hat{U}_0(w) - \hat{U}_L(w)}{\hat{U}_H(w) - \hat{U}_L(w)}$$

This cutoff gives rise to the ex ante probabilities of observing the date-0 outcome x_0, x_H and x_L. I denote them by $\hat{p}^0(w)$,$\hat{p}^H(w)$ and $\hat{p}^L(w)$ respectively. This gives two more constraints for the principal's problem:

$$\hat{p}^0(w)\hat{U}_0 + \hat{p}^H(w)\hat{U}_H + \hat{p}^L(w)\hat{U}_L - V \geq \theta_3$$
$$\hat{p}^0(w)\hat{U}_0 + \hat{p}^H(w)\hat{U}_H + \hat{p}^L(w)\hat{U}_L - V \geq \max\{\hat{U}_0, 0.5\hat{U}_H + 0.5\hat{U}_L\}$$

The above constraints ensure that the agent accepts the contract and subsequently chooses to work rather than shirk at date-0, given that he will choose according to (4.6) at date-1.

Thus, the principal solves the following problem:

$$\max_{w\in\mathcal{R}^9_+} \hat{p}^0(w)\hat{X}_0(w) + \hat{p}^H(w)\hat{X}_H(w)\hat{p}^L(w)\hat{X}_L(w) \tag{4.7}$$

subject to

for every $i \in \{0, H, L\}$

$$p_{i0}w_{i0} + p_{iH}w_{iH} + p_{iL}w_{iL} - V \geq \max\{w_{i0}, 0.5w_{iH} + 0.5w_{iL}\} \tag{4.8}$$

$$\hat{p}^0(w)\hat{U}_0 + \hat{p}^H(w)\hat{U}_H + \hat{p}^L(w)\hat{U}_L - V \geq \theta_3 \tag{4.9}$$

$$\hat{p}^0(w)\hat{U}_0 + \hat{p}^H(w)\hat{U}_H + \hat{p}^L(w)\hat{U}_L - V \geq \max\{\hat{U}_0, 0.5\hat{U}_H + 0.5\hat{U}_L\} \tag{4.10}$$

Suppose the principal offers the agent a linear contract: $w_{ij} = \alpha_1 x_i + \alpha_2 x_j + \beta$, where $\alpha_1, \alpha_2 \geq 0$ and $\beta \in \mathcal{R}$. Since the contract is linear the date-1 cutoff becomes p_f for all date-0 outcomes. Further, the ex ante probabilities $p^{i0}(w)$, $p^{iH}(w)$ and $p^{iL}(w)$ become independent of i. In particular, they become p^0_f, p^H_f, p^L_f. Irrespective of what the agent does at date-0, at date-1 he will get $E_f X$. As a consequence, the date-0 cutoff also becomes p_f. The principal's problem is as follows:

$$\max_{\alpha_1,\alpha_2\geq 0,\beta\in\mathcal{R}} (2 - \alpha_1 - \alpha_2)E_f X - \beta \tag{4.11}$$

subject to

$$\alpha_2(E_f X - \max\{(0.5x_H + 0.5x_L), x_0\}) \geq V \tag{4.12}$$

$$(\alpha_1 + \alpha_2)E_f X + \beta \geq 2V + \theta_3 \tag{4.13}$$

$$\alpha_1(E_f X - \max\{(0.5x_H + 0.5x_L), x_0\}) \geq V \tag{4.14}$$

$$(\alpha_1 + \alpha_2)x_L + \beta \geq 0 \tag{4.15}$$

It is possible to restrict attention to contracts in which $\alpha_1 = \alpha_2$ without loss of generality. To see this, consider an arbitrary optimal contract,$(\alpha_1^*, \alpha_2^*, \beta^*)$, and define $\alpha_1^{`} = \alpha_2^{`} = \frac{\alpha_1^*+\alpha_2^*}{2}$. The contract $(\alpha_1^{`}, \alpha_2^{`}, \beta^*)$ yields the same payoff to the principal as $(\alpha_1^*, \alpha^*, \beta^*)$ and satisfies all the constraints. Thus, it is also optimal. With this in mind, I set $\alpha_1 = \alpha_2$, the objective function becomes:

$$(2 - 2\alpha)E_f X - \beta$$

and constraint set becomes:

$$\begin{aligned}\alpha(E_f X - \max\{(0.5x_H + 0.5x_L), x_0\}) &\geq V \\ 2\alpha E_f X + \beta &\geq 2V + \theta_3 \\ 2\alpha x_L + \beta &\geq 0\end{aligned}$$

Let $\beta' = \frac{\beta}{2}$ and $\theta_3' = \frac{\theta_3}{2}$. Then, the above equations become:

$$(1 - \alpha)E_f X - \beta'$$

and

$$\begin{aligned}\alpha(E_f X - \max\{(0.5x_H + 0.5x_L), x_0\}) &\geq V \\ \alpha E_f X + \beta' &\geq V + \theta_3' \\ \alpha x_L + \beta' &\geq 0\end{aligned}$$

This is the same problem as (4.1)-(4.4). There exists an optimal contract that is linear and implements the first best if:

$$\frac{V(E_{UI}X - x_L)}{\pi_I} \leq \theta_3' = \frac{\theta_3}{2} \tag{4.16}$$

If the parameters of the problem satisfy (4.16) then in any optimal contract the principal pays the agent an expected wage of $2V + \theta_3$ and the agent chooses the cutoff p_f. Notice, that if $\min\{\theta_1, \theta_2\} < \frac{\theta_3}{2}$, for example if $\theta_3 = \theta_1 + \theta_2$, then for parameter values such that:

$$\min\{\theta_1, \theta_2\} < \frac{V(E_{UI}X - x_L)}{\pi_I} \leq \frac{\theta_3}{2}$$

the principal would be able to use a linear contract implementing the cutoff p_f in both periods and paying $2V + \theta_3$ if he used two-period contracting, but not if he used period-by-period contracting.

This result is not affected by observability. That is to say, suppose we restrict what the principal observes at the end of date-1 to the sum of the output rather than being able to tell when each quantity was realized, the result is unchanged. This is because the optimal linear contract with equal alphas depends only on the sum of the output in each period.

4.4 Two-period contracting with recontracting

Finally, I study the case where the agent can recontract at date-1. That is, at the end of date-1 the agent can decide whether to stay with the principal or take his outside option θ_2. The limited liability constraint is imposed on the total wage paid out to the agent, rather than his period-by-period payout. This is done to maintain consistency and comparability with the previous case.

Suppose the principal offers the agent $w_{ij} = \alpha_1 x_i + \alpha_2 x_j + \beta_1 + \beta_2$, where $\alpha_1, \alpha_2 \geq 0$ and $\beta_1, \beta_2 \in \mathcal{R}$. I assume that if the agent chooses to leave at date-1, he receives $\alpha_1 x_i + \beta_1 + \theta_2$, where x_i is the date-0 outcome.

The principal's problem with recontracting is identical to the problem in the previous section with two exceptions. First, there is an additional participation constraint - the date-1 participation constraint. Second, the outside option of the agent at date-0 is $\theta_1 + \theta_2$, rather than θ_3. The problem is given below:

$$\max_{\alpha_1,\alpha_2 \geq 0, \beta_1, \beta_2 \in \mathcal{R}} (2 - \alpha_1 - \alpha_2) E_f X - \beta_1 - \beta_2 \tag{4.17}$$

subject to

$$\alpha_2 E_f X + \beta_2 \geq V + \theta_2 \tag{4.18}$$

$$\alpha_2 (E_f X - \max\{(0.5 x_H + 0.5 x_L), x_0\}) \geq V \tag{4.19}$$

$$(\alpha_1 + \alpha_2) E_f X + \beta_1 + \beta_2 \geq 2V + \theta_1 + \theta_2 \tag{4.20}$$

$$\alpha_1 (E_f X - \max\{(0.5 x_H + 0.5 x_L), x_0\}) \geq V \tag{4.21}$$

$$(\alpha_1 + \alpha_2) x_L + \beta_1 + \beta_2 \geq 0 \tag{4.22}$$

The first two constraints ensure that the agent remains with the principal at date-1 and chooses to work. The subsequent two constraints guarantee that the agent accept the contract and chooses to work at date-0. The last constraint is the limited liability constraint.

By an argument analogous to the one in the previous section, I can restrict my attention to the special case where $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ without loss of generality. The objective function can be written as:

$$2(1-\alpha)E_f X - 2\beta$$

and the constraints can be written as:

$$\alpha E_f X + \beta \geq V + \theta_2 \tag{4.23}$$

$$\alpha(E_f X - \max\{(0.5x_H + 0.5x_L), x_0\}) \geq V \tag{4.24}$$

$$\alpha E_f X + \beta \geq V + \frac{\theta_1 + \theta_2}{2} \tag{4.25}$$

$$\alpha(E_f X - \max\{(0.5x_H + 0.5x_L), x_0\}) \geq V \tag{4.26}$$

$$\alpha x_L + \beta \geq 0 \tag{4.27}$$

Inequalities (4.24) and (4.26) are identical and one of them can be dropped. If $\theta_1 \geq \theta_2$, then $\frac{\theta_1+\theta_2}{2} \geq \theta_2$. Inequality (4.25) implies (4.23) and the above constraints can be written as:

$$\alpha E_f X + \beta \geq V + \frac{\theta_1 + \theta_2}{2}$$

$$\alpha(E_f X - \max\{(0.5x_H + 0.5x_L), x_0\}) \geq V$$

$$\alpha x_L + \beta \geq 0$$

These constraints are identical to (4.2)-(4.4) and thus there exists an optimal contract that is linear and attains the first best if

$$\frac{V(E_{UI}X - x_L)}{\pi_I} \leq \frac{\theta_1 + \theta_2}{2}$$

If $\theta_1 < \theta_2$, then $\frac{\theta_1+\theta_2}{2} < \theta_2$ then inequality (4.23) implies (4.25) and there exists an optimal contract that is linear and attains the first best if

$$\frac{V(E_{UI}X - x_L)}{\pi_I} \leq \theta_2$$

Thus, there exists an optimal contract that implements the first best if

$$\frac{V(E_{UI}X - x_L)}{\pi_I} \leq \max\{\theta_2, \frac{\theta_1 + \theta_2}{2}\} \tag{4.28}$$

Since $\max\{\frac{\theta_1+\theta_2}{2}, \theta_2\} > \min\{\theta_1, \theta_2\}$, we have that recontracting allows for a wider range of parameter values under which an efficient outcome can be implemented. That is, if

$$\min\{\theta_1, \theta_2\} < \frac{V(E_{UI}X - x_L)}{\pi_I} \leq \max\{\theta_2, \frac{\theta_1 + \theta_2}{2}\}$$

the principal would be able to use a linear contract implementing the cutoff p_f in both periods and paying $2V + \max\{2\theta_2, \theta_1 + \theta_2\}$ if he used two-period contracting, but not if he used period-by-period contracting.

The expected payoff to the agent from the contract with recontracting is $2V + \max\{2\theta_2, \theta_1 + \theta_2\}$, which is greater than or equal to the corresponding payoff from period-by-period contracting, $2V + \theta_1 + \theta_2$. That raises the threshold value of $\frac{V(E_{UI}X - x_L)}{\pi_I}$ until which the first best can be implemented.

4.5 Conclusion

Contracting over multiple periods can under some circumstances improve the efficiency of the outcome. In this chapter, I compare three contracting regimes - period-by-period contracting, two-period contracting without recontracting and two-period contracting with recontracting. I find that two-period contracting with recontracting expands the range of parameters under which the first-best outcome can be implemented (using a linear contract).

The key take away from this chapter is that certain contracting regimes raise the expected payoff to the agent without the actual outside options being higher. This raises the upper bound for the set of parameters under which the first best can be implemented. The choice of contracting regime depends on the trade off between the minimum expected payoff under the participation constraint and the loss of payoff induced by not using a linear contract.

While this essay sheds light upon the trade off between cost and risk taking in a dynamic framework, it does not shed light on the characteristics of the optimal contract. In the static framework, contracts could be linear, concave or convex because of the fact that there were only 3 outcomes. In the dynamic framework, the principal can choose from a richer set of contracts. A fruitful extension of this analysis would be to shed light on the particular features of the two-period or multiperiod contract.

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Appendix A

Appendix to chapter 2

A.1 Conditions for separating equilibria

Given below are the conditions that need to hold for separating actions to be be an equilibrium.

If kkkf or kkff then

$2(\theta_X^1 + \theta_Y^1 - 1) \leq \phi_Y$

If kfkf or kfff then

$(2 - \theta_Y^2)(\theta_X^1 + \theta_Y^1 - 1) \leq \phi_Y$

A.2 Conditions for pooling equilibria

Given below are the conditions that need to hold for different pooling actions to be equilibria.

1. Pooling at X

   (a) $\phi_Y \leq \theta_X^2 + \theta_Y^2 - 1$

   (b) If $u_y(X) + \theta_x^2 u_y(Y) + (1 - \theta_x^2)\tau_y - 2\tau_y > 0$ then
   $\phi_Y \leq \frac{(1+\theta_X^2)(\theta_X^1+\theta_Y^1-1)}{\theta_X^1+\theta_Y^1}$

(c) If $u_y(X) + \theta_x^2 u_y(Y) + (1-\theta_x^2)\tau_y - 2\tau_y \leq 0$ then

No pooling

2. Pooling at Y

(a) $\phi_X \leq \theta_X^2 + \theta_Y^2 - 1$

(b) If $u_x(Y) + \theta_Y^2 u_x(X) + (1-\theta_y^2)\tau_x - 2\tau_x > 0$

$\phi_X \leq \frac{(1+\theta_Y^2)(\theta_X^1+\theta_Y^1-1)}{\theta_X^1+\theta_Y^1}$

(c) If $u_x(Y) + \theta_Y^2 u_x(X) + (1-\theta_y^2)\tau_x - 2\tau_x \leq 0$

No pooling

3. Pooling at XY

(a) $\phi_X \leq \theta_X^2 + \theta_Y^2 - 1$

(b) $\phi_Y \leq (1+\theta_Y^2)(\theta_X^1 + \theta_Y^1 - 1)$

(c) If $u_x(Y) + \theta_Y^2 u_x(X) + (1-\theta_y^2)\tau_x - 2\tau_x < 0$

$\phi_X \geq \frac{(1+\theta_Y^2)(\theta_X^1+\theta_Y^1-1)}{\theta_X^1+\theta_Y^1}$

(d) If $u_x(Y) + \theta_Y^2 u_x(X) + (1-\theta_y^2)\tau_x - 2\tau_x \geq 0$

a and b are sufficient.

4. Pooling at YX

(a) $\phi_Y \leq \theta_X^2 + \theta_Y^2 - 1$

(b) $\phi_X \leq (1+\theta_X^2)(\theta_X^1 + \theta_Y^1 - 1)$

(c) If $u_y(X) + \theta_x^2 u_y(Y) + (1-\theta_x^2)\tau_y - 2\tau_y < 0$ then

$\phi_Y \geq \frac{(1+\theta_X^2)(\theta_X^1+\theta_Y^1-1)}{\theta_X^1+\theta_Y^1}$

(d) If $u_y(X) + \theta_x^2 u_y(Y) + (1-\theta_x^2)\tau_y - 2\tau_y \geq 0$ then

a and b are sufficient

Appendix B

Appendix to chapter 3

B.1 Lemmas and Proofs

The two lemmas that follow attempt to establish the relationship between μ_1^*, μ_2^* and η^*. Both lemmas are used later for establishing the relationship between p_f and p_s.

Lemma B.1. *If $\eta^* \neq 0$ then at least one of μ_1^* and μ_2^* are strictly greater than zero. If $\eta^* = 0$ then either both μ_1^* and μ_2^* are zero or they are both strictly greater than 0.*

Proof. When $\eta^* \neq 0$. Now suppose both μ_1^* and μ_2^* are zero. Equations (3.22) and (3.23) become:

$$1 = \lambda^* + \eta^* \frac{1}{p_0^*}$$
$$1 = \lambda^* - \eta^* \frac{p_s}{p_H^*}$$

This implies that

$$\eta^* \frac{1}{p_0^*} = -\eta^* \frac{p_s}{p_H^*}$$

$$\eta^* \frac{0.5(1-p_s^2)+p_s^2}{p_0^* p_H^*} = 0$$
$$\eta^* \frac{0.5(1+p_s^2)}{p_0^* p_H^*} = 0$$

Which cannot be true for $\eta^* \neq 0$.

When $\eta^* = 0$, equations (3.22) and (3.23) become

$$\mu_1^* \frac{p_0^*-1}{p_0^*} + \mu_2^* = \mu_1^* + \mu_2^* \frac{p_H^*-0.5}{p_H^*}$$
$$\mu_1^* (\frac{p_0^*-1-p_0^*}{p_0^*}) = \mu_2^* (\frac{p_H^*-0.5-p_H^{*\,*}}{p_H})$$
$$\mu_1^* (\frac{1}{p_0^*}) = \mu_2^* (\frac{0.5}{p_H^*})$$
$$\mu_1^* (\frac{1}{p_s}) = \mu_2^* (\frac{1}{1-p_s^2})$$

We know that $p_s \neq 1$. Thus, either both μ_1^* and μ_2^* are zero or they are both strictly greater than 0. □

Lemma B.2. *a) Suppose $\eta^* \neq 0$. If $p_s > 0.5$, then $\mu_1^* > 0$, $\mu_2^* = 0$ and $\eta^* > 0$. If $p_s < 0.5$, then $\mu_1^* = 0$, $\mu_2^* > 0$ and $\eta^* < 0$.*

b) Suppose $\eta^* = 0$. If $p_s \neq 0.5$, then $\mu_1^* = \mu_2^* = 0$.

Proof. If $p_s > 0.5$, then by definition of the cutoff value, when $p = 0.5$, the safe project yields more than the risky project i.e. $0.5w_H^* + 0.5w_L^* < w_0^*$. This implies that the right hand side of constraint (3.18) is strictly greater than the right hand side of constraint (3.19). Thus, constraint (3.19) must hold with strict inequality and $\mu_2^* = 0$. Then by proposition B.1 $\mu_1^* > 0$ if $\eta^* \neq 0$ and $\mu_1^* = 0$ if $\eta^* = 0$. Further, in the case when $\eta^* \neq 0$, from (3.22) and (3.23) we have that

$$\begin{aligned}
\mu_1^* \frac{p_0^* - 1}{p_0^*} + \eta^* \frac{1}{p_0^*} &= \mu_1^* - \eta^* \frac{p_s}{p_H^*} \\
\frac{p_0^* - 1 - p_0^*}{p_0^*} \mu_1^* &= -\eta^* (\frac{p_s}{p_H^*} + \frac{1}{p_0^*}) \\
\frac{-1}{p_0^*} \mu_1^* &= -\eta^* (\frac{p_s^2 + 0.5(1 - p_s^2)}{p_H^* p_O^*}) \\
\frac{1}{p_0^*} \mu_1^* &= \eta^* (\frac{0.5(1 + p_s^2)}{p_H^* p_O^*})
\end{aligned}$$

which can only be true if $\eta^* > 0$.

Similar argument applies for $p_s < 0.5$.

□

Lemma B.1 is similar to Lambert's proposition 2 and lemma B.2 is similar to Lambert's proposition 3.

Next, I prove some technical lemmas about η^*, p_s and p_f. Equation (3.25) can be rearranged and written as:

$$\Sigma x_j \frac{\partial p_j}{\partial \hat{p}}|_{\hat{p}=p_s} = \Sigma w_j^* \frac{\partial p_j}{\partial \hat{p}}|_{\hat{p}=p_s} + \eta^* (w_H^* - w_L^*)$$

From (3.20) we know that the first term on the right hand side is 0. Thus,

$$\Sigma x_j \frac{\partial p_j}{\partial \hat{p}}|_{\hat{p}=p_s} = \eta^* (w_H^* - w_L^*) \tag{B.1}$$

Further, as argued earlier $w_H^* > w_L^*$. Therefore, the sign of η^* must be equal to the sign of $\Sigma x_j \frac{\partial p_j}{\partial \hat{p}}|_{\hat{p}=p_s}$. From lemma B.2, if $\eta^* \neq 0$, the derivative of the expected value of the project with respect to the project selection cutoff $\hat{p}$ is strictly negative if $p_s < 0.5$ and strictly negative if $p_s > 0.5$.

Lemma B.3. *a)* $\eta^* < 0$ *if and only if* $p_s > p_f$.

b) $\eta^* > 0$ *if and only if* $p_s < p_f$

c) $\eta^* = 0$ *if and only if* $p_s = p_f$

Proof. a) Consider the function $\Sigma x_j p_j = \hat{p}x_0 + 0.5(1-\hat{p}^2)x_H + (1-\hat{p})^2 x_L$. The first and second derivatives of this function are given by

$$\Sigma x_j \frac{\partial p_j}{\partial \hat{p}} = x_0 - (\hat{p}x_H + (1-\hat{p})x_L), \quad \Sigma x_j \frac{\partial^2 p_j}{\partial \hat{p}^2} = x_L - x_H < 0$$

Thus the function is strictly concave. Further, we know that $\Sigma x_j \frac{\partial p_j}{\partial \hat{p}}|_{\hat{p}=p_f} = 0$ since the expected payoff from learning p is maximized at p_f. Thus, for any $\hat{p} = p_s$

$$\Sigma x_j \frac{\partial p_j}{\partial \hat{p}}|_{\hat{p}=p_f} > \Sigma x_j \frac{\partial p_j}{\partial \hat{p}}|_{\hat{p}=p_s} \iff p_s > p_f$$

From (B.1)

$$0 > \eta(w_H^* - w_L^*) \iff p_s > p_f$$

As argued earlier, $w_H^* > w_L^*$. Thus, sign of η is equal to the sign of $p_f - p_s$
b) and c) Same argument as for part a)

□

Now I prove the main result of this paper.

Proposition 3.2. a) If $p_f < 0.5$ then $p_s \in [p_f, 0.5]$.

b) If $p_f > 0.5$ then $p_s \in [0.5, p_f]$.

b) If $p_f = 0.5$ then $p_s = p_f$.

Proof. a) If $p_f < 0.5$ then we have the following possibilities: (i) $p_s > 0.5$, (ii) $p_s < p_f$ or (iii) $p_s \in [p_f, 0.5]$. If $p_s > 0.5 > p_f$, then $p_f - p_s < 0$ and by lemma B.3, $\eta^* < 0$. However, this contradiction lemma B.2. If instead, $p_s < p_f < 0.5$, then by lemma (B.3), $\eta^* > 0$. This again contradicts lemma B.2.Thus, $p_s \in [p_f, 0.5]$.

b) Similar argument as for part (a).

c) If $p_f = 0.5$, then we have the following possibilities: (i) $p_s > 0.5$, (ii) $p_s < 0.5$ or (iii) $p_s = 0.5$. Suppose $p_s > 0.5 = p_f$, then by lemma B.3 $\eta^* < 0$. But this contradicts lemma B.2. Similarly, suppose $p_s < 0.5 = p_f$, then by lemma B.3 $\eta^* > 0$. But this contradicts lemma B.2.

□

In order to prove proposition 3.3 I rewrite the first order conditions (3.22)-(3.24) as follows:

$$\lambda^* = 1 - \xi^* \geq 0 \tag{B.2}$$

$$\mu_1^* = \frac{1}{(1-p_s)^2}(-p_s^2\mu_2^* + (1+p_s)^2\xi^*) \geq 0 \tag{B.3}$$

$$\mu_2^* \geq 0 \tag{B.4}$$

$$\eta^* = \frac{1}{(1-p_s)}(-p_s\mu_2^* + (1-p_s)\xi^*) \tag{B.5}$$

$$\xi^* \geq 0 \tag{B.6}$$

Lemma B.4. *Suppose $p_s \neq 0.5$. Then $\eta^* \neq 0$ if and only if $\xi^* > 0$.*

Proof. Suppose $\xi^* > 0$ and $\eta^* = 0$. Using lemma B.2, $\mu_1^* = \mu_2^* = 0$ and thus from (B.3) we have that $\xi^* = 0$, which is a contradiction. Suppose $\eta^* \neq 0$, then either $\mu_1^* > 0, \mu_2^* = 0$ or $\mu_1^* = 0, \mu_2^* > 0$. Once again, using (B.3), it must be that $\xi^* > 0$. $\square$

The above lemma along with lemma B.2 gives the following proposition.

Proposition 3.3 Suppose $p_s \neq 0.5$. $p_s \neq p_f$ if and only if the multiplier on the limited liability constraint is strictly positive.

Proof. Lemma B.2 and B.4 imply the result. $\square$

Corollary 3.1 If $\frac{V}{\theta} > \frac{\pi_I}{(E_{UI}X - x_L)}$ and $p_f \neq 0.5$, then the optimal contract is not linear.

Proof. To see this, notice that if the optimal linear contract was optimal in general then $p_s = p_f$. Using proposition 3.3 $p_s = p_f$ implies that $\xi^* = 0$. Using (B.2), we then have that $\lambda^* > 0$ and the participation constraint must bind with equality, which is untrue. $\square$

B.2 Concavity of objective function

consider the objective function (3.16) of formulation 1. Let $S_i = x_i - w_i$ be the surplus that accrues to the principal when the output is x_i. The Hessian matrix of the objective function is:

$$H = \begin{pmatrix} -S_H + S_L & -1 & p & (1-p) \\ -1 & 0 & 0 & 0 \\ p & 0 & 0 & 0 \\ (1-p) & 0 & 0 & 0 \end{pmatrix}$$

The Hessian is not negative semidefinite. Thus, the objective function is not concave.

Next, I check if it is quasiconcave. In order to do so, I calculate the bordered Hessian matrices which are given below:

$$D_1 = \begin{pmatrix} 0 & S_0 - (pS_H + (1-p)S_L) \\ S_0 - (pS_H + (1-p)S_L) & -S_H + S_L \end{pmatrix}$$

$$D_2 = \begin{pmatrix} 0 & S_0 - (pS_H + (1-p)S_L) & -p \\ S_0 - (pS_H + (1-p)S_L) & -S_H + S_L & -1 \\ -p & -1 & 0 \end{pmatrix}$$

$$D_3 = \begin{pmatrix} 0 & S_0 - (pS_H + (1-p)S_L) & -p & -\frac{1}{2}(1-\hat{p}^2) \\ S_0 - (pS_H + (1-p)S_L) & -S_H + S_L & -1 & p \\ -p & -1 & 0 & 0 \\ -\frac{1}{2}(1-\hat{p}^2) & p & 0 & 0 \end{pmatrix}$$

$$D_4 = \begin{pmatrix} 0 & S_0 - (pS_H + (1-p)S_L) & -p & -\frac{1}{2}(1-\hat{p}^2) & -\frac{1}{2}(1-\hat{p})^2 \\ S_0 - (pS_H + (1-p)S_L) & -S_H + S_L & -1 & p & (1-p) \\ -p & -1 & 0 & 0 & 0 \\ -\frac{1}{2}(1-\hat{p}^2) & p & 0 & 0 & 0 \\ -\frac{1}{2}(1-\hat{p})^2 & (1-p) & 0 & 0 & 0 \end{pmatrix}$$

$|D_1| < 0$, but $|D_2| \ngtr 0$. So, the objective function is not even quasiconcave.